▶ SAM SANDERS, *On the computability theory and reverse mathematics of the uncountable*.
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I provide an overview of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable ([2–7]). In particular, we have shown that the following theorems are *hard to prove* relative to the usual scale of (higher-order) comprehension axioms, while the objects claimed to exist by these theorems are similarly *hard to compute*, in the sense of Kleene's S1-S9 schemes ([1]).

1. There is no injection from $\mathbb{R}$ to $\mathbb{N}$ (Cantor 1874).
2. Arzelà's convergence theorem for the Riemann integral (1885).
3. A Riemann integrable function is not everywhere discontinuous (Hankel, 1870).
4. Jordan's decomposition theorem.
5. A function of bounded variation on the unit interval has a point of continuity.
6. A lower semi-continuous function on the unit interval attains its minimum.
7. The Bolzano-Weierstrass theorem for countable sets (injections/bijections to $\mathbb{N}$)

We show that the final item gives rise to many robust equivalences, which in turn yields the 'Big Six' and 'Big Seven' system of Kohlenbach's higher-order Reverse Mathematics.

We discuss how comprehension is unsuitable as a measure of logical and computational strength in this context; we also provide an alternative, namely a (classically valid) continuity axiom from Brouwer's intuitionistic mathematics. In turn, this yields a framework that marries the 'best of both worlds' of the Turing and Kleene approach to computability theory.

Finally, our study shows that coding practise common in e.g. Reverse Mathematics and Weihrauch reducibility can significantly change the logical and computational strength of basic theorems pertaining to functions of bounded variation and other 'close to full continuity' notions.

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