When is the Scott topology countably based?

I) Introduction

Scott topology

X

topological space

$\downarrow$

$\mathcal{O}(X)$ + Scott topology

I) Introduction

Scott topology

X

topological space

↓

$\mathcal{O}(X)$ + Scott topology

Scott open set

$\mathcal{U} \subseteq \mathcal{O}(X)$ is Scott open iff

- $\mathcal{U}$ is upward closed
- if $(U_i)_{i \in I} \in \mathcal{O}(X)$ and $\bigcup_{i \in I} U_i \in \mathcal{U}$ then there exists J a finite subset of I such that $\bigcup_{j \in J} U_j \in \mathcal{U}$

Scott topology

X
represented space

↓

$\mathcal{O}(X)$ + Scott topology
admissibly represented

Scott open set

$\mathcal{U} \subseteq \mathcal{O}(X)$ is Scott open iff

- $\mathcal{U}$ is upward closed
- if $(U_i)_{i \in I} \in \mathcal{O}(X)$ and $\bigcup_{i \in I} U_i \in \mathcal{U}$ then there exists J a finite subset of I such that $\bigcup_{j \in J} U_j \in \mathcal{U}$

I) Introduction

Scott topology

X
represented space

↓

$\mathcal{O}(X)$ + Scott topology
admissibly represented

used in many proof techniques

enables to characterise properties of X

Scott open set

$\mathcal{U} \subseteq \mathcal{O}(X)$ is Scott open iff

- $\mathcal{U}$ is upward closed
- if $(U_i)_{i\in I} \in \mathcal{O}(X)$ and $\bigcup_{i\in I} U_i \in \mathcal{U}$ then there exists J a finite subset of I such that $\bigcup_{j\in J} U_j \in \mathcal{U}$

I) Introduction

Countably based space

X
represented space

$\mathcal{O}(X)$ + Scott topology
admissibly represented

used in many proof techniques

enables to characterise properties of X

Y
countably based T0-space

Scott open set

$\mathcal{U} \subseteq \mathcal{O}(X)$ is Scott open iff

- $\mathcal{U}$ is upward closed
- if $(U_i)_{i \in I} \in \mathcal{O}(X)$ and $\bigcup_{i \in I} U_i \in \mathcal{U}$ then there exists J a finite subset of I such that $\bigcup_{j \in J} U_j \in \mathcal{U}$

I) Introduction

Countably based space

X
represented space

$\mathcal{O}(X)$ + Scott topology
admissibly represented

used in many proof techniques

enables to characterise properties of X

Y
countably based T0-space

where the theory of represented spaces was first developed (*Weihrauch*)

where descriptive complexity theory works the best (*De Brecht*, *Yamamoto*, *Callard & Hoyrup*)

many natural spaces

Scott open set

$\mathcal{U} \subseteq \mathcal{O}(X)$ is Scott open iff

- $\mathcal{U}$ is upward closed
- if $(U_i)_{i \in I} \in \mathcal{O}(X)$ and $\bigcup_{i \in I} U_i \in \mathcal{U}$ then there exists J a finite subset of I such that $\bigcup_{j \in J} U_j \in \mathcal{U}$

I) Introduction
The question

X
represented space

$\mathcal{O}(X)$ + Scott topology
admissibly represented

used in many proof techniques

enables to characterise properties of X

Y
countably based T0-space

where the theory of represented spaces was first developed (*Weihrauch*)

where descriptive complexity theory works the best (*De Brecht*, *Yamamoto*, *Callard & Hoyrup*)

many natural spaces

Given X admissibly represented when is $\mathcal{O}(X)$ countably based ?

I) Introduction
Existing results

X admissibly represented **Hausdorff** $\Longrightarrow$ (*Schröder*)

$$\left(\begin{array}{c} \mathcal{O}(X) \text{ countably based} \end{array} \iff \begin{array}{c} X_n \xrightarrow{Lim} X \\ X_n \text{ compact metrisable} \\ X_n \subseteq X_{n+1} \end{array} \iff \begin{array}{c} X \text{ admits a countable pseudobase of compact sets} \end{array} \right)$$

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*)

$$\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$$

previously known?

II) Inductive limit

Hausdorff case

X admissibly represented **Hausdorff** $\Longrightarrow$ (*Schröder*) $\left(\mathcal{O}(X) \text{ countably based } \Longleftrightarrow \begin{array}{l} X_n \xrightarrow{Lim} X \\ X_n \text{ compact metrisable} \\ X_n \subseteq X_{n+1} \end{array} \right)$

Inductive limit

Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of topological spaces. Then $X_n \xrightarrow{Lim} X$ iff

- $X = \bigcup_{n\in\mathbb{N}} X_n$
- $U \in \mathcal{O}(X)$ iff for all n, $U \cap X_n \in \mathcal{O}(X_n)$

II) Inductive limit
Non-Hausdorf case

$$\underset{\text{admissibly represented}}{X} \Longrightarrow \left(\underset{\text{countably based}}{\mathcal{O}(X)} \overset{?}{\Longleftarrow} \begin{array}{c} X_n \xrightarrow{Lim} X \\ \dots \end{array} \right)$$

Inductive limit

Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of topological spaces. Then $X_n \xrightarrow{Lim} X$ iff

- $X = \bigcup_{n\in\mathbb{N}} X_n$
- $U \in \mathcal{O}(X)$ iff for all n, $U \cap X_n \in \mathcal{O}(X_n)$

II) Inductive limit

Non-Hausdorf case

X admissibly represented $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based $\Longleftarrow$ $X_n \xrightarrow{Lim} X$, X_n admissibly represented, $\mathcal{O}(X_n)$ countaby based $\Big)$

Inductive limit

Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of topological spaces. Then $X_n \xrightarrow{Lim} X$ iff

- $X = \bigcup_{n\in\mathbb{N}} X_n$
- $U \in \mathcal{O}(X)$ iff for all n, $U \cap X_n \in \mathcal{O}(X_n)$

II) Inductive limit

Non-Hausdorf case

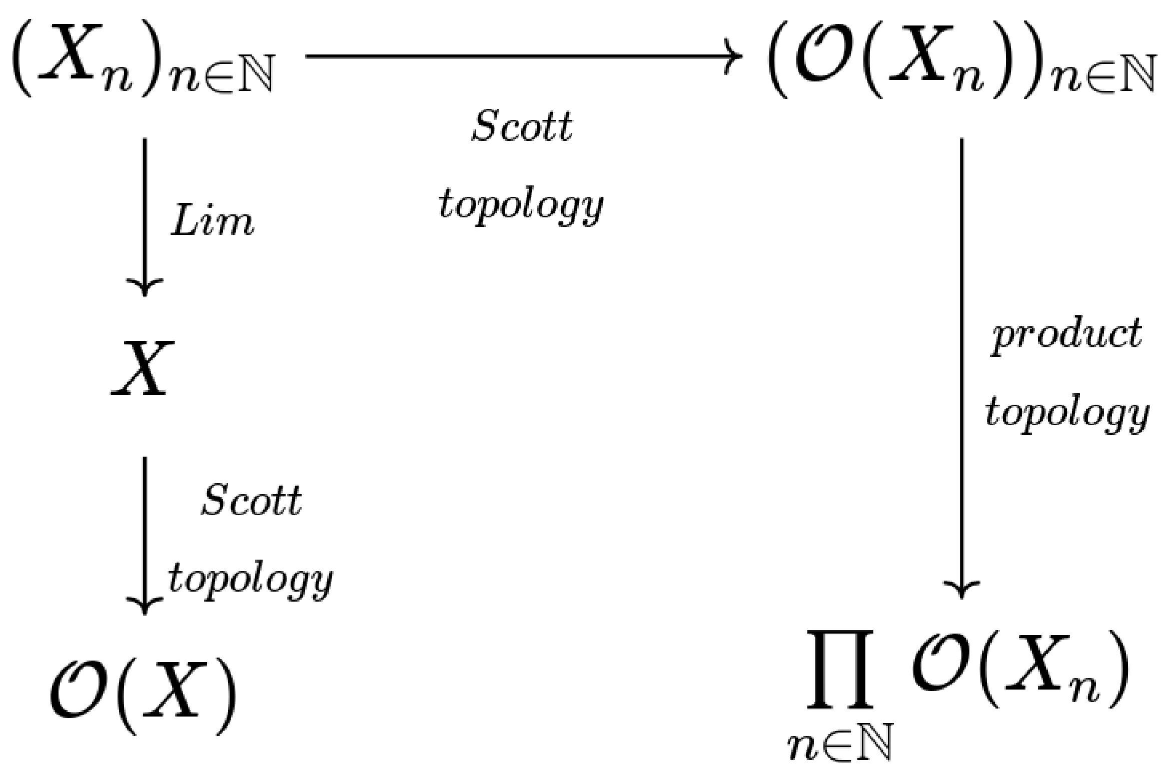
X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\Longleftarrow$ $X_n \xrightarrow{Lim} X$, X_n admissibly represented, $\mathcal{O}(X_n)$ countaby based)

$$\begin{array}{ccc}
(X_n)_{n\in\mathbb{N}} & \xrightarrow[\text{Scott topology}]{} & (\mathcal{O}(X_n))_{n\in\mathbb{N}} \\
\downarrow {\scriptstyle Lim} & & \\
X & & \\
\downarrow {\scriptstyle Scott\ topology} & & \\
\mathcal{O}(X) & &
\end{array}$$

II) Inductive limit

Non-Hausdorf case

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\Longleftarrow$ $X_n \xrightarrow{Lim} X$, X_n admissibly represented, $\mathcal{O}(X_n)$ countaby based)



II) Inductive limit
Non-Hausdorf case

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\Longleftarrow$ $X_n \xrightarrow{Lim} X$ admissibly represented, X_n admissibly represented, $\mathcal{O}(X_n)$ countaby based)

$$\begin{array}{ccc}
(X_n)_{n\in\mathbb{N}} & \xrightarrow[\text{Scott topology}]{} & (\mathcal{O}(X_n))_{n\in\mathbb{N}} \\
\downarrow Lim & & \\
X & & \downarrow \text{product topology} \\
\downarrow \text{Scott topology} & & \\
\mathcal{O}(X) & \xrightarrow[\text{embedding}]{} & \prod_{n\in\mathbb{N}} \mathcal{O}(X_n)
\end{array}$$

II) Inductive limit

Non-Hausdorf case

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\Longleftarrow$ $X_n \xrightarrow{Lim} X$, X_n admissibly represented, $\mathcal{O}(X_n)$ countaby based)

$$
\begin{array}{ccc}
(X_n)_{n\in\mathbb{N}} & \xrightarrow[\text{topology}]{\text{Scott}} & (\mathcal{O}(X_n))_{n\in\mathbb{N}} \\
\downarrow \scriptstyle{Lim} & & \\
X & & \downarrow \scriptstyle{\text{product topology}} \\
\downarrow \scriptstyle{\text{Scott topology}} & & \\
\mathcal{O}(X) & \xrightarrow[\text{embedding}]{} & \prod_{n\in\mathbb{N}} \mathcal{O}(X_n)
\end{array}
$$

II) Inductive limit

Canonical representation

$$X_n \xrightarrow{Lim} X$$

X_n admissibly represented

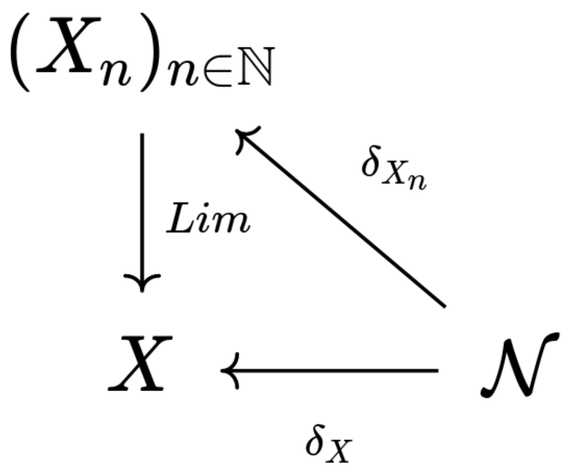
$$(X_n)_{n\in\mathbb{N}}$$
$$\Big\downarrow Lim$$
$$X$$

II) Inductive limit

Canonical representation

$$X_n \xrightarrow{Lim} X$$

X_n admissibly represented

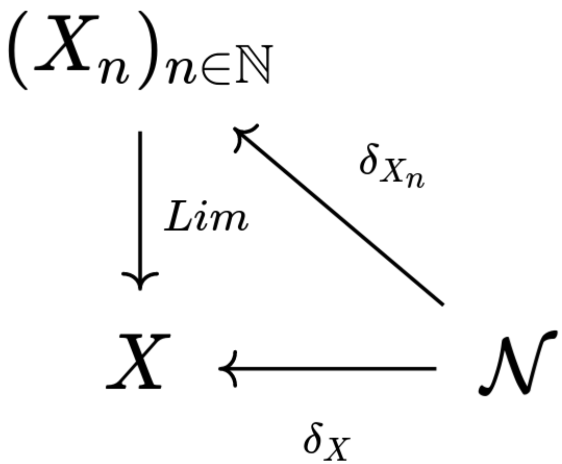


II) Inductive limit

Canonical representation

$$X_n \xrightarrow{Lim} X$$

X_n admissibly represented



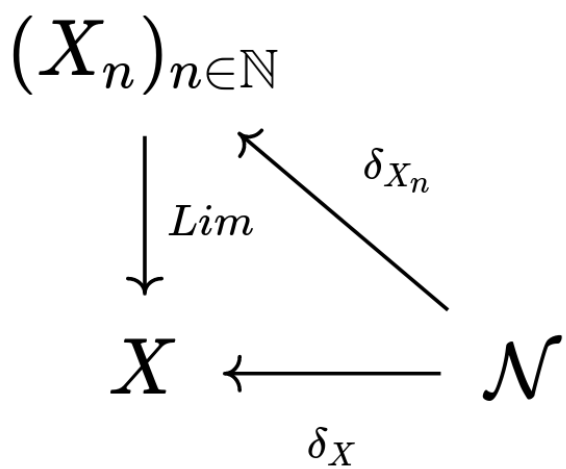
$$x \in X_n$$

name of x by δ_X $=$ n :: name of x by δ_{X_n}

II) Inductive limit

Canonical representation

δ_X admissible $\Longleftarrow$ (*Schröder*) $X_n \xrightarrow{Lim} X$; X_n admissibly represented; X $T1$



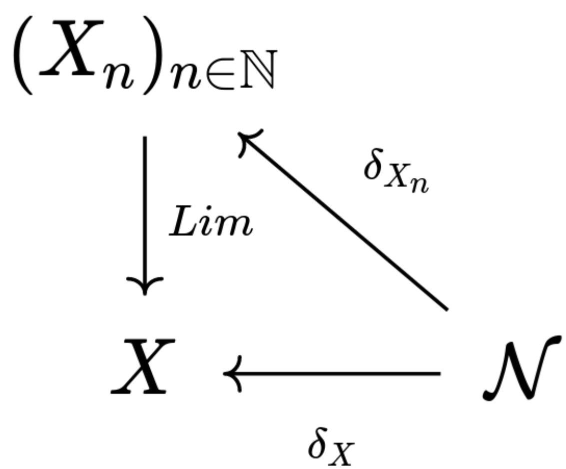
$$x \in X_n$$

name of x by δ_X $=$ n :: name of x by δ_{X_n}

II) Inductive limit

Canonical representation

δ_X admissible $\Longleftarrow$ $X_n \xrightarrow{Lim} X$, X_n admissibly represented, X_n upward closed in X_{n+1}



$$x \in X_n$$

name of x by δ_X $=$ n :: name of x by δ_{X_n}

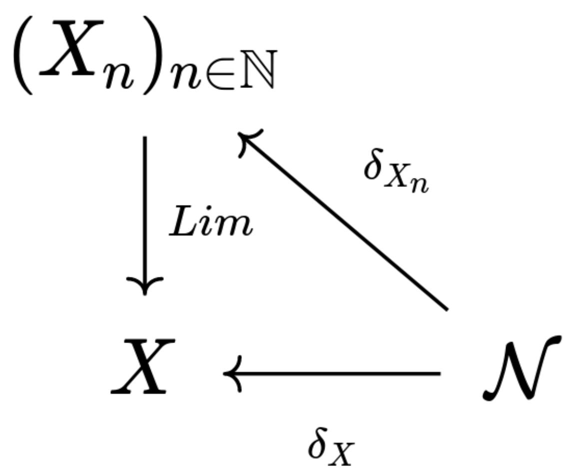
II) Inductive limit

Canonical representation

δ_X admissible $\Longleftarrow$ $X_n \xrightarrow{Lim} X$

X_n admissibly represented

X_n upward closed in X_{n+1}



$$x \in X_n$$

name of x by δ_X $=$ n :: name of x by δ_{X_n}

III) Countable pseudobase of compact sets

Hausdorff case

X admissibly represented **Hausdorff** $\Longrightarrow$ (*Schröder*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

III) Countable pseudobase of compact sets

Hausdorff case

X admissibly represented **Hausdorff** $\Longrightarrow$ (*Schröder*) $\left(\mathcal{O}(X) \text{ countably based} \Longleftarrow X \text{ admits a countable pseudobase of compact sets} \right)$

III) Countable pseudobase of compact sets

Sufficient condition…

X admissibly represented $\underset{\text{(\textit{Schröder})}}{\Longrightarrow}$ $\Big($ $\mathcal{O}(X)$ countably based $\Longleftarrow$ X admits a countable pseudobase of compact sets $\Big)$

III) Countable pseudobase of compact sets

Sufficient condition…

X admissibly represented $\underset{(\textit{Schröder})}{\Longrightarrow}$ $\left(\mathcal{O}(X) \text{ countably based} \Longleftarrow X \text{ admits a countable pseudobase of compact sets} \right)$

Y countably based $\Longrightarrow$ $\mathcal{O}(\mathcal{O}(Y))$ countably based

III) Countable pseudobase of compact sets

Sufficient condition…

X admissibly represented $\overset{\text{(Schröder)}}{\Longrightarrow}$ $\Big($ $\mathcal{O}(X)$ countably based $\Longleftarrow$ X admits a countable pseudobase of compact sets $\Big)$

Y countably based $\Longrightarrow$ $\mathcal{O}(Y)$ admits a countable pseudobase of compact sets

Y countably based $\Longrightarrow$ $\mathcal{O}(\mathcal{O}(Y))$ countably based

III) Countable pseudobase of compact sets

Sufficient condition…

X admissibly represented $\Longrightarrow$ (Schröder) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftarrow$ X admits a countable pseudobase of compact sets $\Big)$

Y countably based $\Longrightarrow$ $\mathcal{O}(Y)$ admits a countable pseudobase of compact sets

Y countably based $\Longrightarrow$ $\mathcal{O}(\mathcal{O}(Y))$ countably based

III) Countable pseudobase of compact sets

Sufficient condition…

$$X \implies \left(\mathcal{O}(X) \overset{?}{\implies} X \right)$$

X admissibly represented $\implies$ ($\mathcal{O}(X)$ countably based $\overset{?}{\implies}$ X admits a countable pseudobase of compact sets)

III) Countable pseudobase of compact sets
…but not necessary

$$\underset{\text{admissibly represented}}{X} \implies \left(\underset{\text{countably based}}{\mathcal{O}(X)} \not\Longrightarrow \underset{\text{admits a countable pseudobase of compact sets}}{X} \right)$$

(*Hofmann & Lawson*)

III) Countable pseudobase of compact sets
…but not necessary

X admissibly represented $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

$$I = [0;1]_{euclide}$$

(*Hofmann & Lawson*)

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$$I = [0;1]_{euclide}$$

(*Hofmann & Lawson*)

$$J = [0;1)_{\geq}$$ open sets are $[0;a)$

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

$A \subseteq I = [0;1]_{euclide}$ $\qquad$ A Bernstein set

(*Hofmann & Lawson*)

$J = [0;1)_{\geq}$ $\qquad$ open sets are $[0;a)$

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow \Big($ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Bernstein set

(*Hofmann & Lawson*)

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ A Bernstein set

(*Hofmann & Lawson*)

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ A Bernstein set

(*Hofmann & Lawson*)

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

III) Countable pseudobase of compact sets

…but not necessary

X admissibly represented $\Longrightarrow$ (✓ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ A Bernstein set

(*Hofmann & Lawson*)

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$$

III) Countable pseudobase of compact sets
…but not necessary

X admissibly represented $\Longrightarrow$ (✓ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Bernstein set

(*Hofmann & Lawson*)

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

$(K_n)_{n\in\mathbb{N}}$ pseudobase of compact sets

$\Downarrow$

A Borel

III) Countable pseudobase of compact sets

A less complexe counter-example

X admissibly represented $\Longrightarrow$ (✓ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Bernstein set

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

$(K_n)_{n\in\mathbb{N}}$ pseudobase of compact sets

$\Downarrow$

$A \in \Sigma^0_3$

III) Countable pseudobase of compact sets

A less complexe counter-example

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based ✓ $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

$(K_n)_{n \in \mathbb{N}}$ pseudobase of compact sets

$\Downarrow$

$A \in \Sigma^0_3$

III) Countable pseudobase of compact sets

A less complexe counter-example

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based ✓ $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

$X =$ monochromatic paires $\quad \Delta^0_4$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

$(K_n)_{n\in\mathbb{N}}$ pseudobase of compact sets

$\Downarrow$

$A \in \Sigma^0_3$

III) Countable pseudobase of compact sets

A less complexe counter-example

X admissibly represented	$\Longrightarrow$	($\mathcal{O}(X)$ countably based $\not\Longrightarrow$	X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires Δ^0_4

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

$(K_n)_{n \in \mathbb{N}}$ pseudobase of compact sets

$\Downarrow$

$A \in \Sigma^0_3$

IV) Core-compactness

Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

IV) Core-compactness

Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

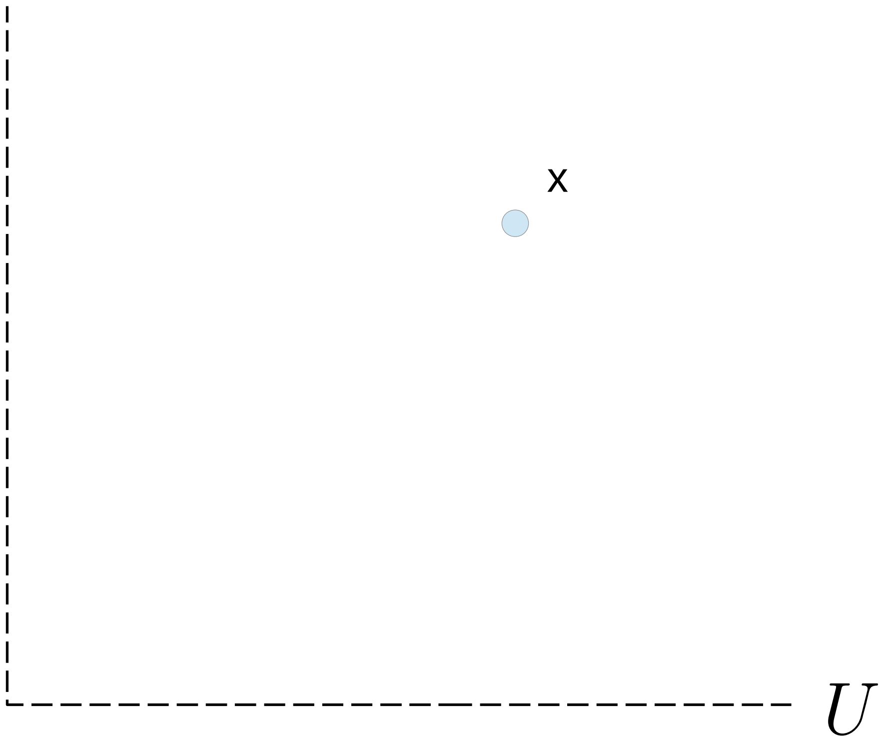
local compactness | **core-compactness**

IV) Core-compactness

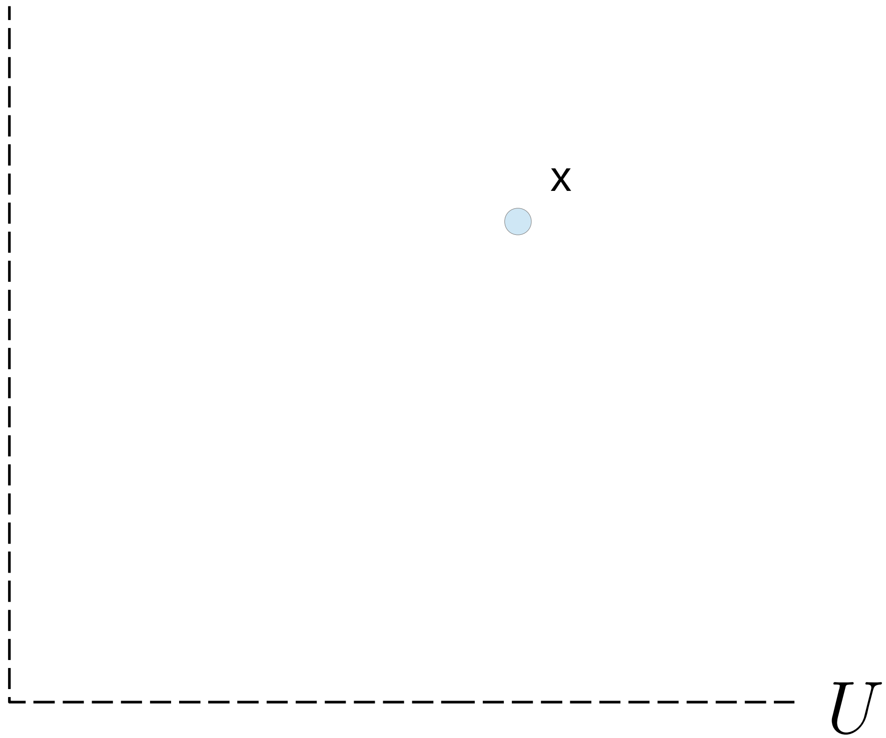
Definition

X admissibly represented **countably based** $\overset{(Hoyrup)}{\Longrightarrow}$ $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

local compactness



core-compactness

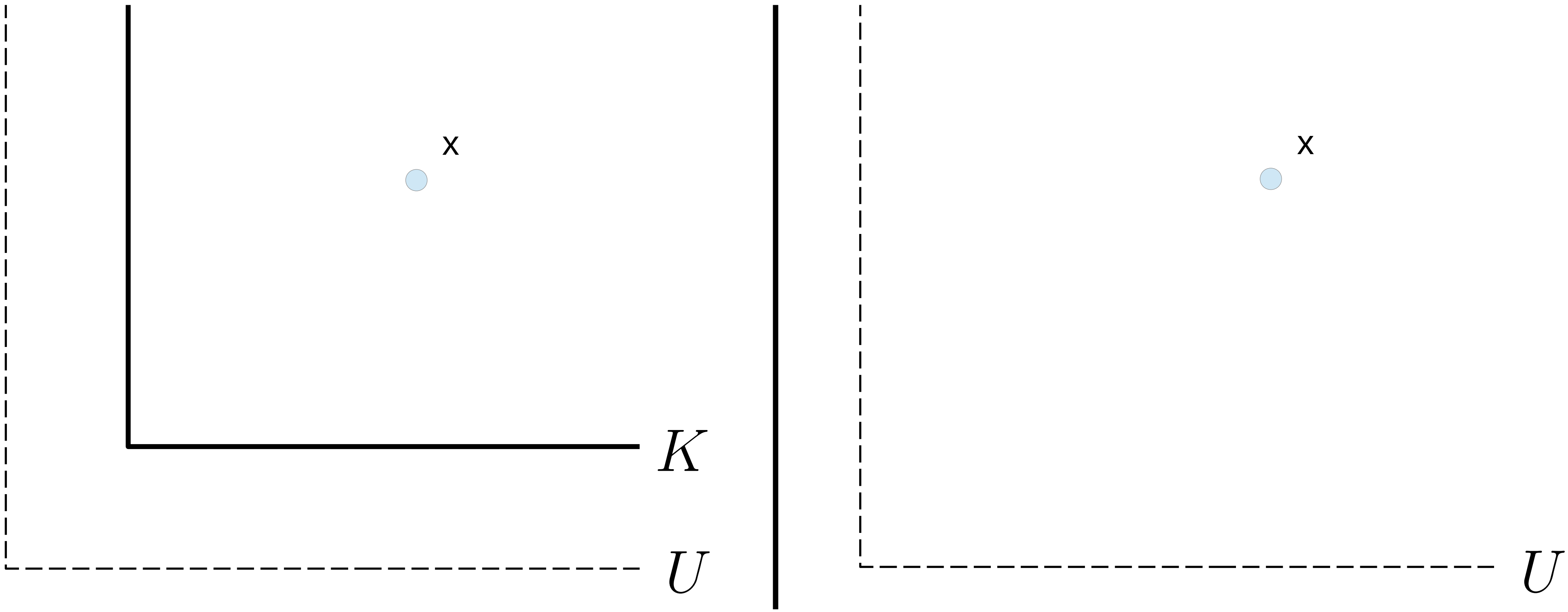


IV) Core-compactness

Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X core-compact $\Big)$

local compactness | **core-compactness**

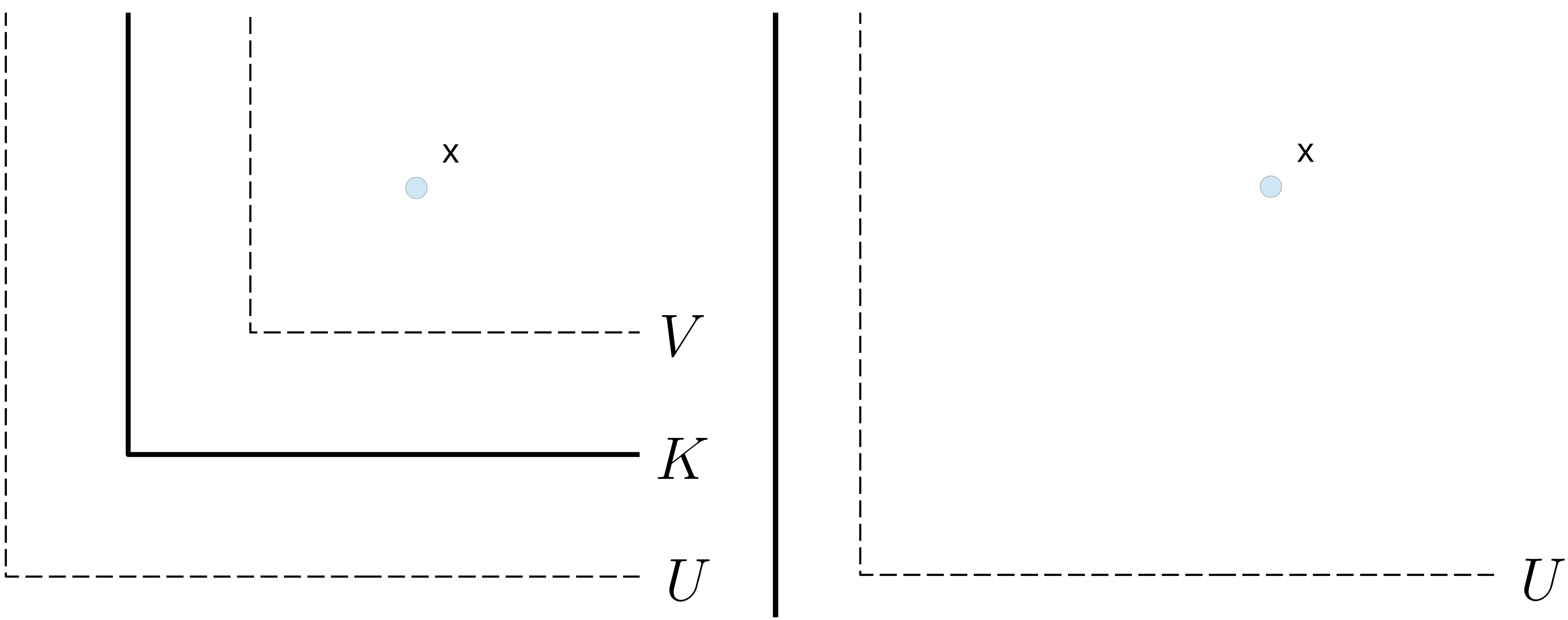


IV) Core-compactness

Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \Longleftrightarrow X \text{ core-compact} \right)$

local compactness | **core-compactness**

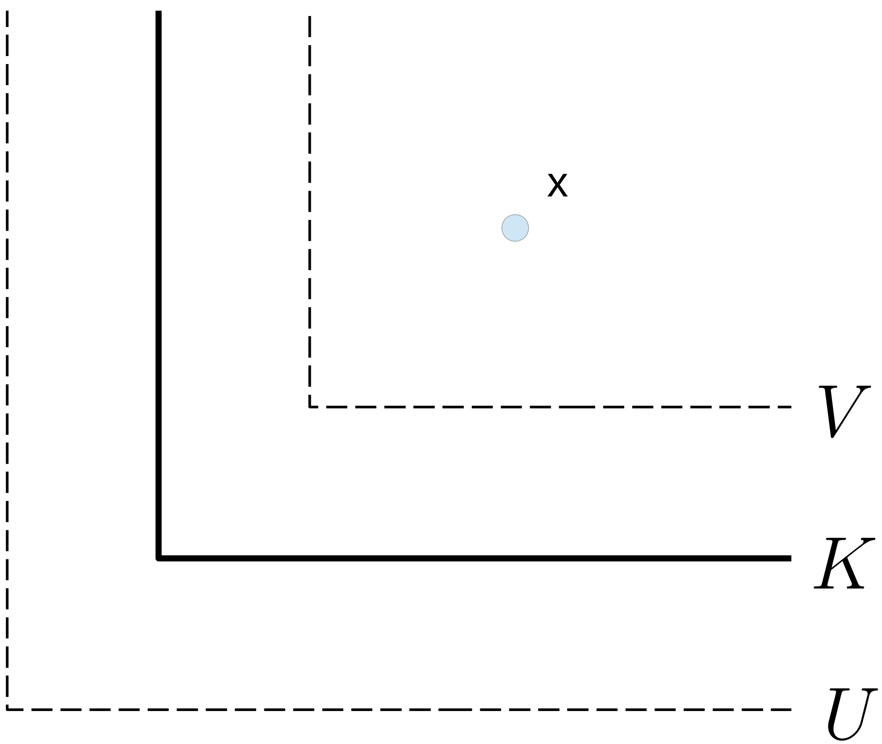


IV) Core-compactness

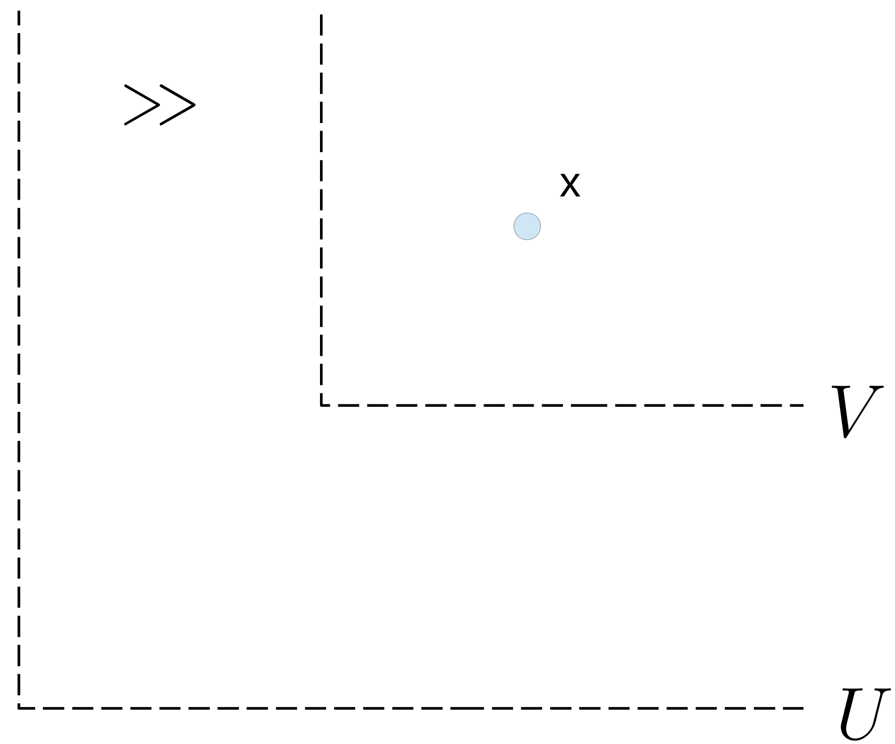
Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X core-compact $\Big)$

local compactness



core-compactness

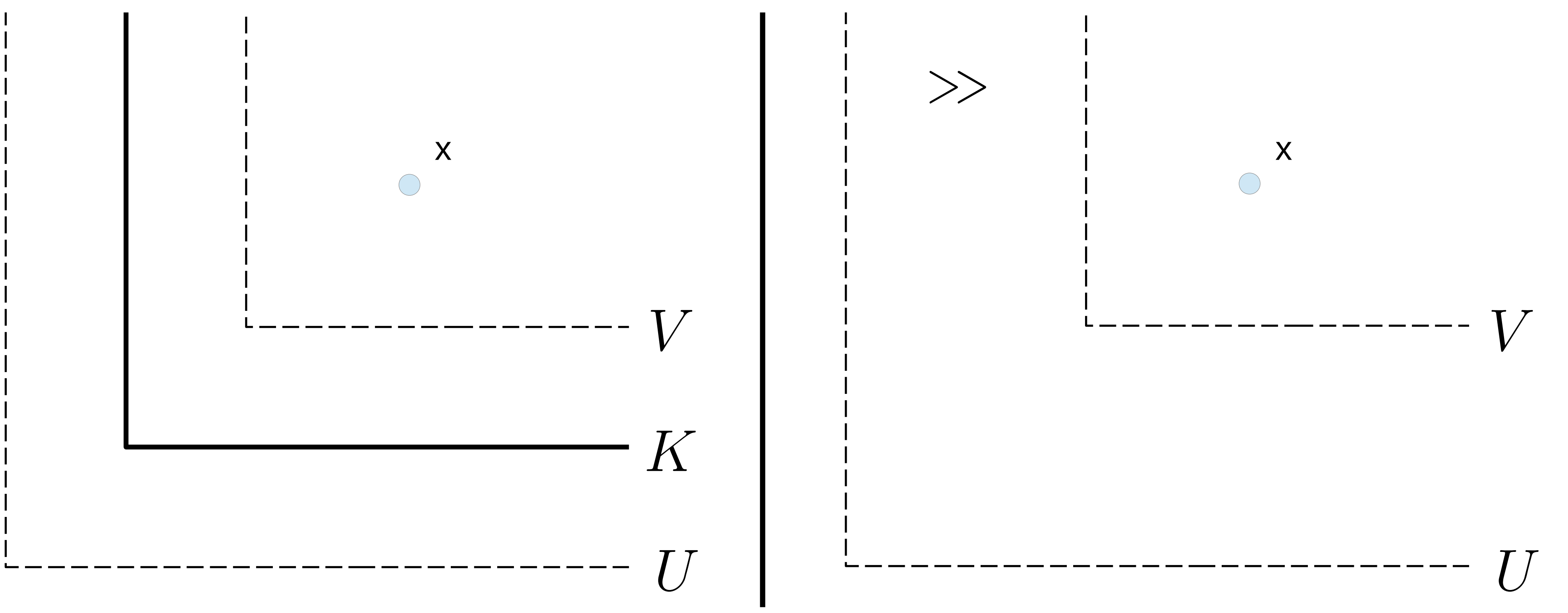


IV) Core-compactness

Definition

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X core-compact $\Big)$

local compactness $\Longrightarrow$ **core-compactness**

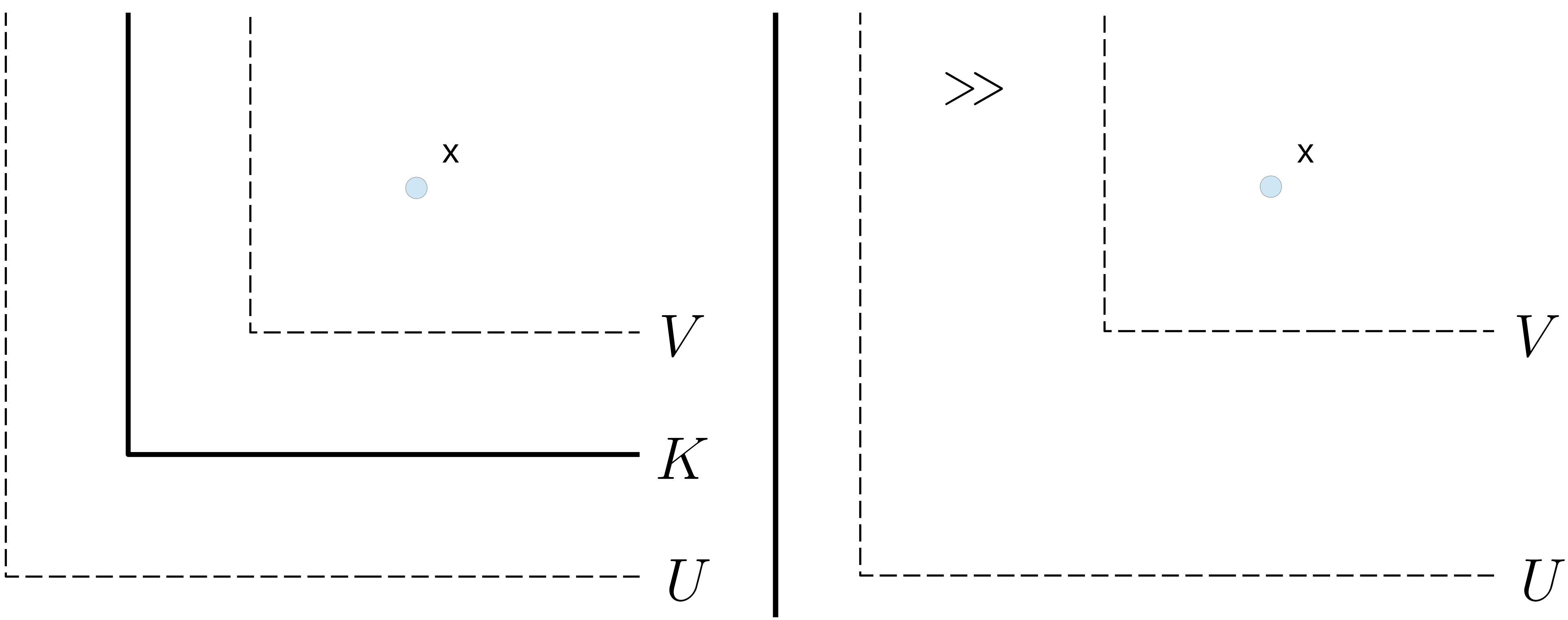


IV) Core-compactness

Counter-exemple

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \Longleftrightarrow X \text{ core-compact} \right)$

local compactness $\not\Longleftarrow$ **core-compactness**



IV) Core-compactness
Counter-exemple

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X core-compact $\Big)$

local compactness $\not\Longleftarrow$ (*Hofmann & Lawson*) **core-compactness**

$A \subseteq I = [0;1]_{euclide}$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

IV) Core-compactness
Counter-exemple

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

local compactness $\not\Longleftarrow$ (*Hofmann & Lawson*) **core-compactness**

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ open sets are $[0;a)$ $\quad$ X non sigma-compact

$X =$ monochromatic paires $\subseteq I \times J$

locally compact countably based implies sigma-compact

$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$

Core-compactness and local compactness

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

local compactness $\iff$ (*Hofmann & Mislove*) **core-compactness**

when X is sober

Sober space

A topological space X is sober iff every irreducible closed subset of X is the closure of a (unique) point.

IV) Core-compactness

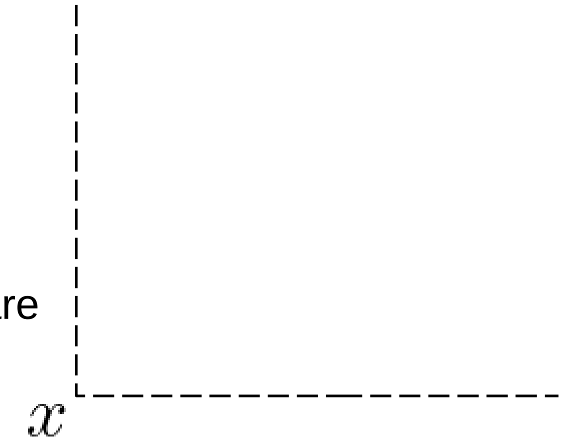
Core-compactness and local compactness

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\Big($ $\mathcal{O}(X)$ countably based $\Longleftrightarrow$ X core-compact $\Big)$

local compactness $\Longleftrightarrow$ **core-compactness**

when $X \subseteq [0;1]^2_{\leq}$

open sets are



where

$x \in [0;1]^2_{\leq}$

IV) Core-compactness

Core-compactness and local compactness

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

local compactness $\iff$ **core-compactness**

when $X \subseteq [0;1]^2_{\leq}$

IV) Core-compactness

Core-compactness and local compactness

X admissibly represented **countably based** $\Longrightarrow$ (*Hoyrup*) $\left(\mathcal{O}(X) \text{ countably based} \iff X \text{ core-compact} \right)$

local compactness $\overset{?}{\iff}$ **core-compactness**

when $X \subseteq [0;1]^n_{\leq}$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented $\Longrightarrow$ ($\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets)

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Π^0_3-*complete*

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

$X =$ monochromatic paires $\subseteq I \times J$

$$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based $\not\Longrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

$A \subseteq I = [0;1]_{euclide}$ $\quad$ A Π^0_3-complete

$\mathbb{Q} \subseteq J = [0;1)_{\geq}$ $\quad$ open sets are $[0;a)$

non-sober $\quad X =$ monochromatic paires $\subseteq I \times J$ $\quad$ **sober**

$$(\mathcal{O}(X), \subseteq) \cong (\mathcal{O}(I \times J), \subseteq)$$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented **sober** $\Longrightarrow \Big($ $\mathcal{O}(X)$ countably based $\overset{?}{\Longrightarrow}$ X admits a countable pseudobase of compact sets $\Big)$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented **sober** $\Longrightarrow$ $\Big($ $\mathcal{O}(X)$ countably based with filters $\Longleftrightarrow$ X admits a countable pseudobase of compact sets $\Big)$

V) Countable pseudobase of compact sets (cont'd)

Lack of sobriety

X admissibly represented **sober**	$\Longrightarrow$	($\mathcal{O}(X)$ countably based with filters	$\Longleftrightarrow$	X admits a countable pseudobase of compact sets)

Conclusion

when $X \subseteq [0;1]^2_{\leq}$

local compactness $\Longleftrightarrow$ **core-compactness**

X admissibly represented **sober**	$\Longrightarrow$	$\mathcal{O}(X)$ countably based with filters	$\Longleftrightarrow$	X admits a countable pseudobase of compact sets
X admissibly represented; counter-example X Δ^0_4	$\Longrightarrow$	$\mathcal{O}(X)$ countably based	$\not\Longrightarrow$	X admits a countable pseudobase of compact sets
X admissibly represented	$\Longrightarrow$	$\mathcal{O}(X)$ countably based	$\Longleftarrow$	$X_n \xrightarrow{Lim} X$; X_n admissibly represented; $\mathcal{O}(X_n)$ countaby based

canonical δ_X admissible $\Longleftarrow$ $X_n \xrightarrow{Lim} X$; X_n admissibly represented; X_n upward closed in X_{n+1}

Y countably based $\Longrightarrow$ $\mathcal{O}(\mathcal{O}(Y))$ countably based