ORACLE COMPUTABILITY OF CONDITIONAL EXPECTATIONS ONTO SUBFACTORS

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A von Neumann algebra is a unital *-subalgebra of the set of bounded operators on some complex Hilbert space. The commutative von Neumann algebras are all isomorphic to those of the form $L^\infty(X, \mu)$ for some measure space (X, μ) (viewed as multiplication operators on the Hilbert space $L^2(X, \mu)$), whence von Neumann algebra theory is often dubbed "noncommutative" measure theory. The von Neumann algebras with trivial center are called factors and are the basic building blocks of all von Neumann algebras. The infinite-dimensional factors that carry a (faithful, normal) trace, a functional akin to a noncommutative integral, are called II_1 factors.

Given a subfactor N of a II_1 factor M, there is a canonical *conditional expectation* mapping $E : M \to N$ sending an element of M to its nearest (in the sense of the norm induced by the trace) element of N; this mapping is the noncommutative analog of the usual conditional expectation map in probability theory.

Our main question is the following: suppose that D is a Turing degree and N is a subfactor of M. Suppose further that one has presentations of M and N such that both are computable from D and for which the inclusion mapping $N \hookrightarrow M$ is also computable from D. Is the conditional expectation $E : M \to N$ also computable from D?

In this talk, we discuss a positive instance of this question when M is an *existentially closed* II_1 factor (a notion of 'genericity" from model theory) and N is a *property (T)* subfactor (an important *rigidity* notion from operator algebras) which is presented using a special kind of presentation we call a *Kazhdan presentation*.

The material presented here appeared in the paper [1]. We will define all notions mentioned above during the talk.

References

[1] I. Goldbring, *Oracle computability of conditional expectations onto subfactors*, New York Journal of Mathematics **27** (2021), 1085-1095.