Dynamical systems: moving from qualitative to computable results

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In dynamical systems theory there are many problems for which one cannot obtain explicit solutions via some formula. A simple example is obtaining the solution of an initial-value problem defined by an ordinary differential equations. Indeed, even if the problem is defined with the standard functions used in Analysis, its solution might not definable in that way. This example is far from exceptional and exemplifies the limitations of trying to use formulas to solve a given problem. To remediate this situation, several approaches have been used, such as qualitative results (by obtaining qualitative information about the solution, without quantifying it), statistical approaches (obtaining information about "typical" solutions) or using numerical approximations.

Computable analysis provides a powerful framework to characterize solutions in a more general manner, without the limitations inherent to formulas. In particular, it allows the combination of several of the above approaches. In this talk we will explore the following general problem: given a qualitative result, under which circumstances can we obtain a more quantitative (i.e. a computable) version of this result? We will investigate this problem for several classes of problems such as a variant of Hilbert's 16th problem [1]. This talk decribes joint work with N. Zhong.

References

[1] D. S. Graça and N. Zhong. Computing the exact number of periodic orbits for planar flows, 2022. To appear at Transactions of the American Mathematical Society. Preprint available at http://arxiv.org/abs/2101.07701.