Exploring the point-to-set principles for algorithmic dimensions

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Abstract

Effective and resource-bounded dimensions were defined by Lutz in [5] and [4] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [1, 6, 2, 12] and all the references in them).

The point-to-set principle of J. Lutz and N. Lutz [8] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [3, 11] and more recent results in [7, 13, 14, 15]). This characterization has been recently extended to separable spaces [10] and to resource-bounded dimensions [9].

In this talk I will review the point-to-set principles focusing on both the adaptability of algorithmic dimension to different separable spaces and the importance of the oracle that achieves the characterization of classical dimension in terms of an algorithmic dimension. For instance Stull [15] has been able to improve the Marstrand projection theorem by analyzing the optimality of the oracles in the point-to-set principles.

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