COMPUTABLE TOPOLOGY AND TOPOLOGICAL GROUPS

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Abstract. I will talk about several new results in computable topology that enjoy applications of topological and discrete groups. Some of these applications are perhaps unexpected as they involve deep classical results from abstract harmonic analysis and cohomology theory.

Computable topology is a classical field of computable mathematics; the earliest works and ideas concerning computability in the abstract topological sense can be traced back to the 1950s (Malcev, Markov, Specker, and many others). Perhaps, the most well-established notions of computability are: a computable Polish (metrized) space, a computable topological space (usually with c.e. strong inclusion), and an effectively compact space. Obviously, the latter notion applies only in the compact setting, but there are generalisations of the notion to locally compact spaces.

Separating the most natural notions of computable presentability has traditionally been of central importance in computable mathematics. E.g., Markov computable continuous functions vs. Type 2 computable functions on $[0, 1]$; groups with unlovable word problem in combinatorial group theory; c.e. presented vs. computable vs. Π^0_1-presented Boolean algebras, linear orders, and abelian groups in computable algebra, etc. Perhaps unexpectedly, the aforementioned three main effective presentability notions in topology have been separated *only very recently.*

It is perhaps even more unexpected that answering questions of this sort seemingly always requires a wide variety of techniques. In my opinion, it is rather surprising that *computable topological group theory* and *computable (countable) abelian group theory* have recently found non-trivial applications in investigations of this sort. The tight connection between the emerging theory of computable topological groups and the classical subject of computable topology has been *mutually beneficial.* I will therefore also discuss the foundations of computable topological group theory with an emphasis to its connections with computable topology.

The several recent works I will discuss are joint with Downey, Bazhenov, Harrison-Trainor, Lupini, Nies, and Ng; we cite some of these works here: [1, 3, 5]. Of course, the closely related (independent) works will also be mentioned; e.g., [4, 6]. For a detailed exposition of the topic and the machinery used in such investigations, we cite the recent technical survey [2].

References

[1] Nikolay Bazhenov, Matthew Harrison-Trainor, and Alexander Melnikov. Computable Stone spaces, 2021. arXiv:2107.01536.
[2] R. Downey and A. Melnikov. Effectively compact spaces. *Submitted.*
[3] Matthew Harrison-Trainor, Alexander Melnikov, and Keng Meng Ng. Computability of Polish spaces up to homeomorphism. *The Journal of Symbolic Logic*, pages 1–25, 2020.
[4] M. Hoyrup, T. Kihara, and V. Selivanov. Degree spectra of homeomorphism types of Polish spaces. *Preprint*, 2020.
[5] M. Lupini, A. Melnikov, and A. Nies. Computable topological abelian groups. *J. of Algebra, to appear.*
[6] Arno Pauly, Dongseong Seon, and Martin Ziegler. Computing Haar Measures. In volume 152 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 34:1–34:17, Dagstuhl, Germany, 2020.

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