REGAININGLY APPROXIMABLE NUMBERS AND SETS

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Abstract. We call an $\alpha \in \mathbb{R}$ *regainingly approximable* if there exists a computable nondecreasing sequence $(a_n)_n$ of rational numbers converging to α with $\alpha - a_n < 2^{-n}$ for infinitely many $n \in \mathbb{N}$. We also call a c.e. set $A \subseteq \mathbb{N}$ *regainingly approximable* if the strongly left-computable number 2^{-A} is regainingly approximable. We show that the set of regainingly approximable sets is neither closed under union nor intersection and that every c.e. Turing degree contains such a set. Furthermore, the regainingly approximable numbers lie properly between the computable and the left-computable numbers and are not closed under addition. While regainingly approximable numbers are easily seen to be i.o. K-trivial, we construct such an α such that $K(\alpha \upharpoonright n) > n$ for infinitely many n. Similarly, there exist regainingly approximable sets whose initial segment complexity infinitely often reaches the maximum possible for c.e. sets. Finally, there is a uniform algorithm splitting regular real numbers into two regainingly approximable numbers that are still regular.

Keywords: left-computable numbers; effective approximation; computably enumerable sets; splitting; Turing degrees; Kolmogorov complexity

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