Computability of generalized graphs

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In this talk we show that a topological pair of a generalized graph and the set of its endpoints has computable type. Remember that a topological pair (A,B) (i.e. a pair of topological spaces such that $B \subseteq A$) has computable type if

$$S \text{ semicomputable } \Rightarrow S \text{ computable}$$

holds in any computable topological space whenever there exists a homeomorphism $f : A \to S$ such that $f(B)$ is a semicomputable set.

The notion of a generalized graph is inspired by the notion of a graph - a set which consists of finitely many arcs such that distinct arcs intersect in at most one endpoint. The following result was proved in [Iljazović 2020].

Theorem 1 *Let G be a graph and let E be the set of all endpoints of G. Then (G,E) has computable type.*

We define the following: Suppose A is a topological space. Let $V \subseteq A$ be a finite subset of A and let $\mathscr{K}$ be a finite family of pairs $(K,\{a,b\})$ where $a,b \in V$, $a \neq b$ and $K \subseteq A$ is a continuum chainable from a to b. Suppose

$$A = V \cup \bigcup_{(K,\{a,b\}) \in \mathscr{K}} K$$

and that the following holds: if $(K,\{a,b\})$, $(L,\{c,d\}) \in \mathscr{K}$ and $K \neq L$, then $\text{card}(K \cap L) < \aleph_0$. Then the triple $(A,\mathscr{K},V)$ is called a **generalized graph**. See Figure 1.

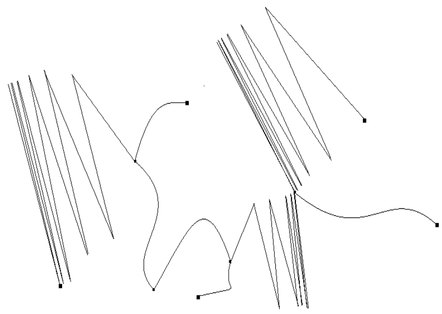

Figure 1: Generalized graph. The highlighted points are its endpoints.

If $(A,\mathscr{K},V)$ is a generalized graph and $a \in V$, we say that a is an **endpoint** of $(A,\mathscr{K},V)$ if there exist only one $K \subseteq A$ and at least one $b \in V$ such that $(K,\{a,b\}) \in \mathscr{K}$.

For such an object, it holds:

Theorem 2 *If $(A,\mathscr{K},V)$ is a generalized graph and B is the set of all its endpoints, then (A,B) has computable type.*

However, by the definition of a generalized graph, we let its edges to intersect in multiple points as long as there are finitely many of them. A question arises naturally - if we allow infinite intersections of edges, can we state an analogous result for such an object. In this talk we give an answer to that question, as well.

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