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The program *Reverse Mathematics* ([8]) can be viewed as a classification of theorems of ordinary, i.e. non-set theoretical, mathematics from the point of view of *computability.* Working in Kohlenbach's *higher-order Reverse Mathematics* ([5]), we study an *alternative classification*, namely based on the central tenet of Feferman's *Explicit Mathematics* ([3, 4]) that:

A proof of existence of an object yields a procedure to compute said object.

To this end, we expand the framework of higher-order Reverse Mathematics with a weak version of Nelson's *internal* approach to Nonstandard Analysis ([1] and [7]). As a result, we obtain a nonstandard version of computability based on infinitesimals, called Ω-invariance, and we can guarantee that standard formulas are decidable (relative to an oracle) *if and only if* they can be transferred (with parameters). In other words, the notions of 'decidable' and 'transferrable' coincide in the standard world ([2]).

Working in the aforementioned framework, we establish the *Explicit Mathematics Theme* (EMT). Intuitively speaking, the EMT states that we can compute an object with certain properties uniformly via a functional *if and only if* said object merely exists *with the same nonstandard properties.* More formally, let T^{st} be a theorem of ordinary mathematics of the form:

$$T^{st} \equiv (\forall^{st} x^\sigma)(A^{st}(x) \to (\exists^{st} y^\tau) B^{st}(x,y)).$$

The ***nonstandard*** version of T^{st} is the statement:

$$T^* \equiv (\forall^{st} x^\sigma)(A^{st}(x) \to (\exists^{st} y^\tau) \boldsymbol{B(x,y)}),$$

where B^{st} is 'transferred' to B, i.e. the standardness predicate 'st' is omitted. Furthermore, the ***uniform*** version of T^{st} is the statement:

$$UT^{st} \equiv \boldsymbol{(\exists^{st} \Phi^{\sigma\to\tau})}(\forall^{st} x^\sigma)(A^{st}(x) \to B^{st}(x, \boldsymbol{\Phi(x)})).$$

The **EMT** is the conjecture that we always have $\boldsymbol{T^* \leftrightarrow UT^{st}}$. We present a number of examples from classical and intuitionistic mathematics, in particular from *proof mining* ([6]). We point out an intimate connection with constructive mathematics.

Finally, the EMT has foundational implications for Hilbert's program, finitism, and structuralism; These will be discussed time-permitting.

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References.

[1] B. van den Berg, E. Briseid, P. Safarik, *A functional interpretation for nonstandard arithmetic*, ***Ann. Pure Appl. Logic***, 163 (2012), no. 12, pp. 1962?1994.

[2] B. van den Berg, S. Sanders, *Transfer equals Comprehension*, ***In preparation***

[3] S. Feferman, *A language and axioms for explicit mathematics, Algebra and logic*, ***Lecture Notes in Math.*** (Fourteenth Summer Res. Inst., Austral. Math. Soc., Monash Univ., Clayton, 1974), Springer, 1975, pp. 87-139. , Vol. 450.

[4] S. Feferman, *Constructive theories of functions and classes*, ***Stud. Logic Foundations Math.*** (LC78, Bergen), vol. 97, North-Holland, 1979, pp. 159-224

[5] U. Kohlenbach, *Higher-order Reverse Mathematics*, ***Lect. Notes Log.***, Reverse Mathematics 2001, vol. 21, Assoc. Symbol. Logic, La Jolla, CA, 2005, pp. 281-295

[6] U. Kohlenbach, *Applied proof theory: proof interpretations and their use in mathematics*, ***Springer Monographs in Mathematics***, Springer-Verlag, Berlin, 2008
[7] E. Nelson, *Internal set theory: a new approach to nonstandard analysis*, ***Bull. Amer. Math. Soc***, 83 (1977), no. 6, 1165-1198
[8] S. Simpson, *Subsystems of Second-order Arithmetic*, ***2nd ed., Perspectives in Logic***, Cambridge University Press, Cambridge, 2009