▸ VITTORIO CIPRIANI, Technische Universität Wien, *On the computational complexity of unfriendly partitions.*

In this talk, we will investigate how difficult it is to compute an *unfriendly partition* of a countable graph. All graphs $G = (V, E)$ we consider are countable, undirected, and without self-loops. A *partition* $V(G) = V_0 \cup V_1$ is *unfriendly* if $(\forall i < 2)$ $(\forall v \in V_i)$ $(|V_i \cap N(v)| \leq |V_{1-i} \cap N(v)|)$, where $N(v) := \{w \in V(G) : (v, w) \in E(G)\}$.

Cowan and Emerson conjectured that every graph has an unfriendly partition, but Milner and Shelah provided a counterexample [5]. However, the graph they exhibit as a counterexample is an uncountable one, therefore, the conjecture remains open for the countable case. It is known that certain classes of graphs admit an unfriendly partition ([1, 3, 2])

In the context of *reverse mathematics*, Shafer [4] showed that ACA_0 is equivalent to the statement "Every locally finite graph has an unfriendly partition".

We start giving some results from the perspective of *Weihrauch reducibility*. Namely, we consider the problem $\mathsf{UP}_{\mathcal{F}}$ for $\mathcal{F}$ being some class of graphs. Such a problem takes a graph in $\mathcal{F}$ as input and outputs an unfriendly partition. We will show that for natural classes of graphs $\mathcal{F}$, $\mathsf{UP}_{\mathcal{F}}$ is Weihrauch equivalent to well-known problems in the Weihrauch lattice.

Finally, we will show that computing unfriendly partitions can be rather complicated, providing an example of a graph such that any unfriendly partition computes any hyperarithmetic set. This talk collects preliminary results from joint works with Belanger, Goh, Jain, Richter and Stephan.

Bibliography

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