▸ THIBAUT KOUPTCHINSKY, *The reverse mathemtaics of analytic measurability.*.
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This talk is about a work in progress with Juan Aguilera (TU Wien) and Keita Yokohama (Tohoku University) on the foundations of mathematics, studying measurability of Σ^1_1 set, with the method of random forcing. Reverse Mathematics is the branch of Mathematical Logic which studies the question: "given a mathematical theorem ϕ, which axioms are necessary to prove ϕ?", therefore providing a comparison of the theorems of mathematics according to some measure of their provability strength (see [2]).

We study a theorem of Lusin [1], which stated that all analytical set of reals are Lebesgue-measurable. We work in the framework of second-order arithmetic. Yu [4] showed that assuming transfinite recursion, ATR_0 is sufficient (and necessary) to prove that $\lambda(A)$ exists for all Borel sets A, coded as wellfounded trees on natural numbers. Simpson [2] asked whether ATR_0 "suffices to prove measurability and regularity of analytic sets in some appropriate sense." The purpose of our work is to answer this question.

The following is our main theorem:

Theorem 1. *The following are equivalent over* ATR_0.

1. *All analytic sets* $A \subset [0,1]$ *are Lebesgue-regular; and*
2. $\boldsymbol{\Sigma}^1_1$*-Induction for* $\mathbb{N}$.

In addition to answering Simpson's question, Theorem 1 presents a reversal of a theorem to $\boldsymbol{\Sigma}^1_1$-Induction. We do not know of any other theorem from core mathematics equivalent to this theory.

Theorem 1 deals with Lebesgue-regularity, i.e., the equality of the outer and inner measures. If one additionally demands that this value exist as a number, the strength of Lusin's theorem increases:

Theorem 2. *The following are equivalent over* ATR_0.

1. *All analytic sets* $A \subset [0,1]$ *are Lebesgue-measurable; and*
2. $\boldsymbol{\Pi}^1_1$*-Comprehension.*

The main idea is to draw inspiration from Solovay's [3] construction of a model of Zermelo-Fraenkel set theory where every set is Lebesgue measurable. In our case the forcing argument is carried out over a non-standard model of a weak set theory (obtained through the familiar method of *pseudohierarchies*). The main subtle point is the use of $\boldsymbol{\Sigma}^1_1$-Induction to guarantee that the forcing provides enough information about a given analytic set A to conclude that $\lambda^*(A) = \lambda_*(A)$.

[1] N. Lusin., *Sur la classification de M. Baire.*, ***Com. Ren. Acad. Sci. Paris*** 164:91–94, 1917.

[2] S. G. Simpson, ***Subsystems of second order arithmetic*** (Second edition), Perspectives in Logic, Association for Symbolic Logic, 2009.

[3] R. Solovay, *A model of set theory in which every set of reals is Lebesgue measurable* ***Ann. Math.*** 92(1):1–56, 1970.

[4] X. Yu, *Riesz representation theorem, Borel measures and subsystems of second-order arithmetic* ***Annals of Pure and Applied Logic*** 59(1):64–78, 1993.