Speedability of computably approximable reals and their approximations

Nan Fang

Abstract

Given a computable approximation $\{a_s\}_{s\in\omega}$ to a real α, we say the approximation is *speedable* if there exists a computable function f such that the modified approximation $\{a_{f(s)}\}_{s\in\omega}$ converges faster than $\{a_s\}_{s\in\omega}$. This leads to various notions of *speedability* for computably approximable reals, depending on the speedability of their computable approximations, and the computational complexity of the reals as well.

Left/right-c.e. reals are those with nondecreasing/nonincreasing computable approximations, respectively; d.c.e. reals are differences of two left-c.e. reals; and computably approximable reals are those that admit a computable approximation. In this talk, we examine speedability notions in the context of these different classes of reals. Previous work by Merkle and Titov established the equivalence of several speedability notions for left-c.e. reals. For right-c.e. reals, the situation is symmetrical. We extend these results to d.c.e. reals, showing that the various notions of speedability coincide in this broader setting as well. Moreover, we prove that for d.c.e. reals, being speedable is equivalent to not being Martin-Löf random. Finally, for the general class of computably approximable reals, we show that every such real admits at least one speedable computable approximation.

This is joint work with George Barmpalias, Wolfgang Merkle, and Ivan Titov.