Computable sets and symmetry points

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Computable sets and rigid points

$S \subseteq \mathbb{R}^n$ computable

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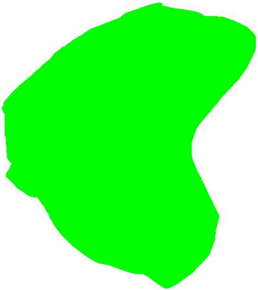

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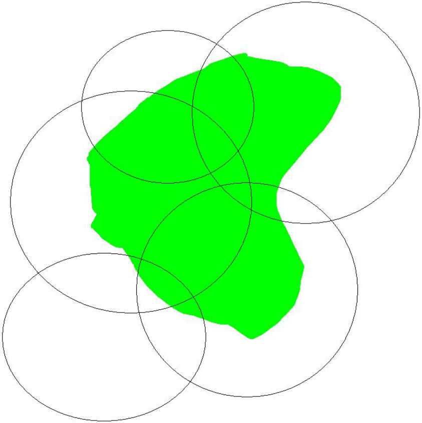

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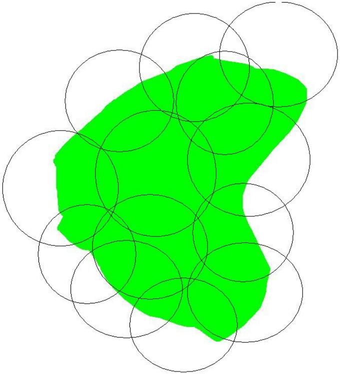

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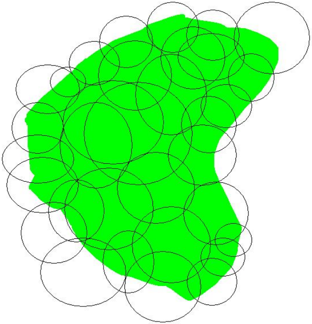

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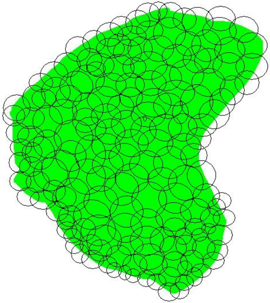

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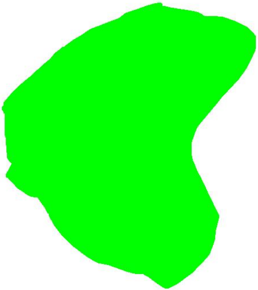

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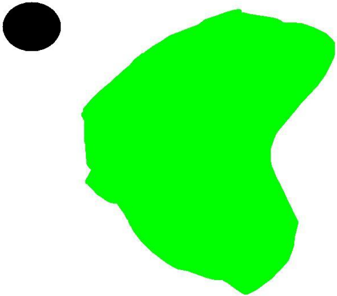

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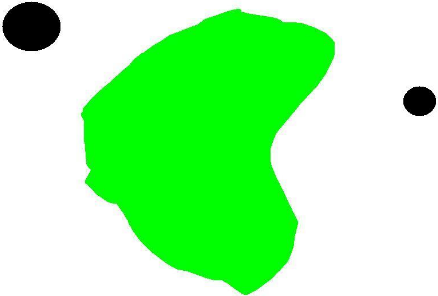

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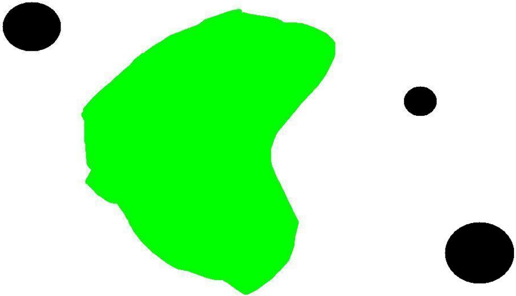

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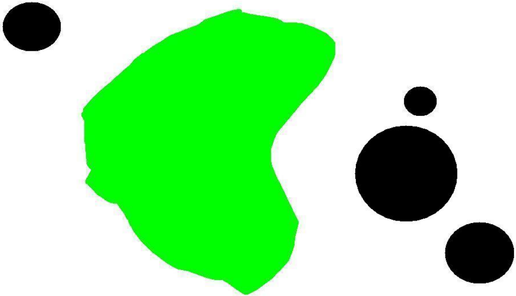

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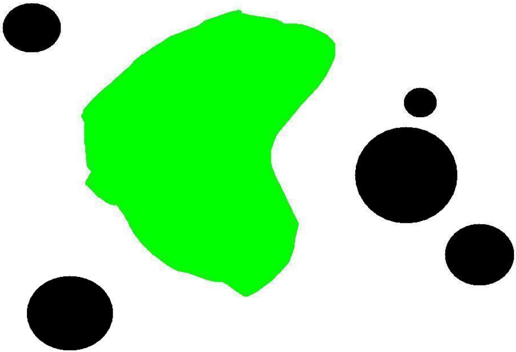

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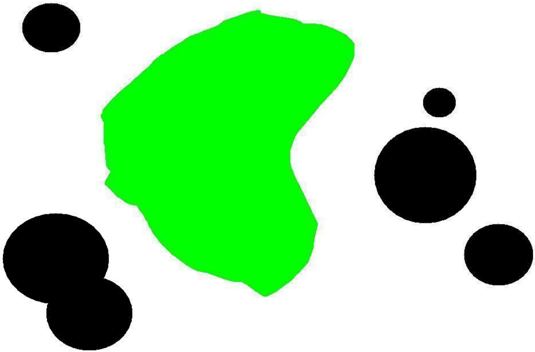

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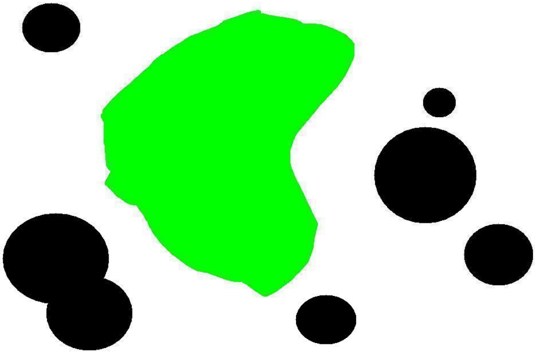

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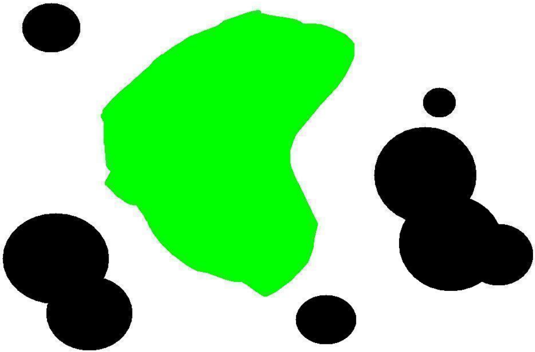

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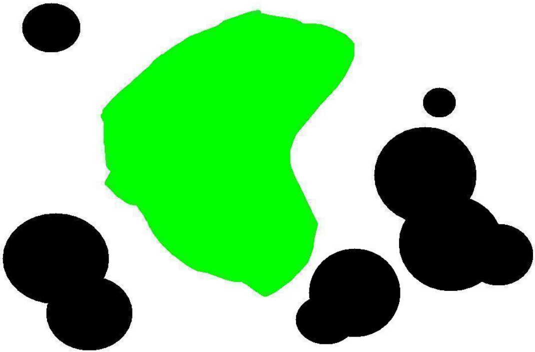

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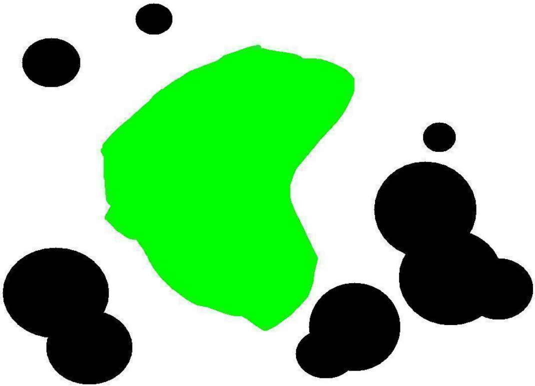

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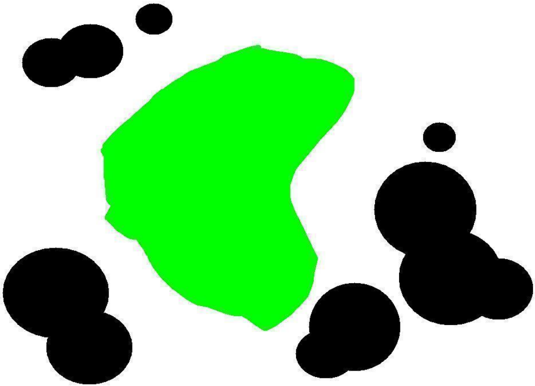

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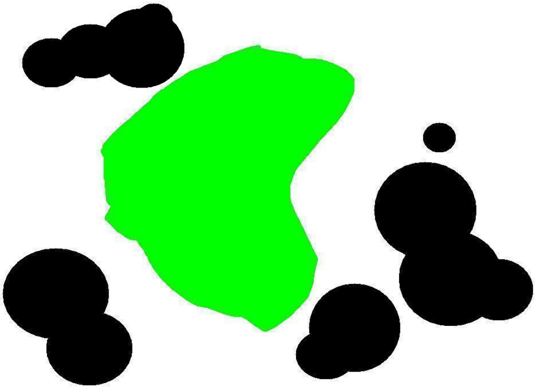

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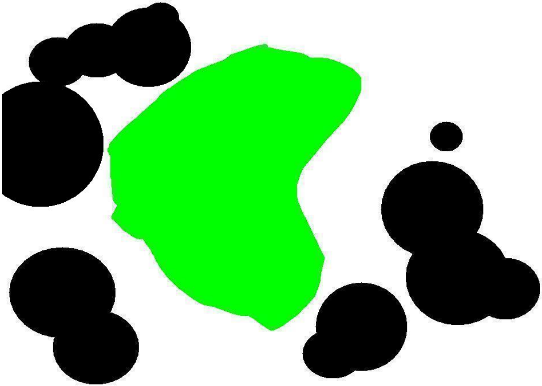

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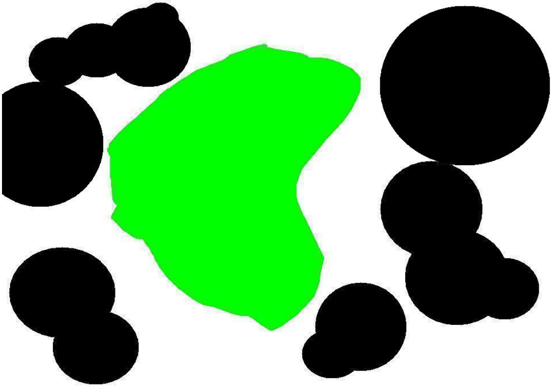

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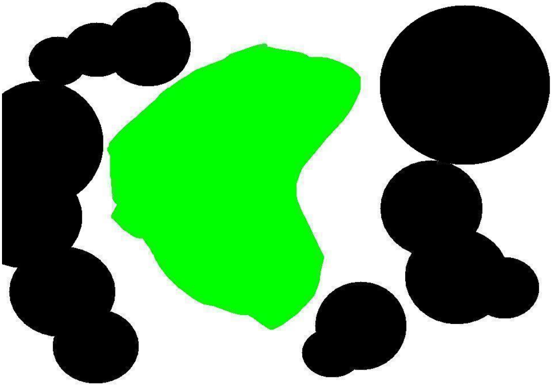

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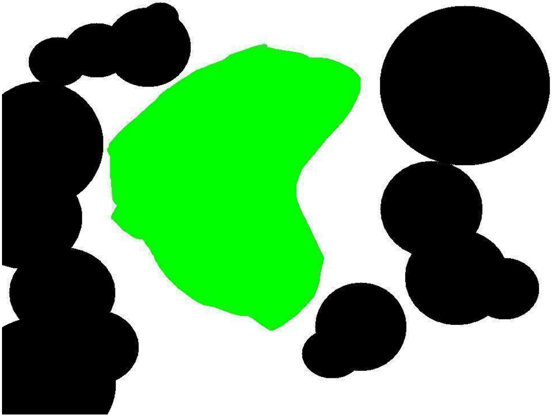

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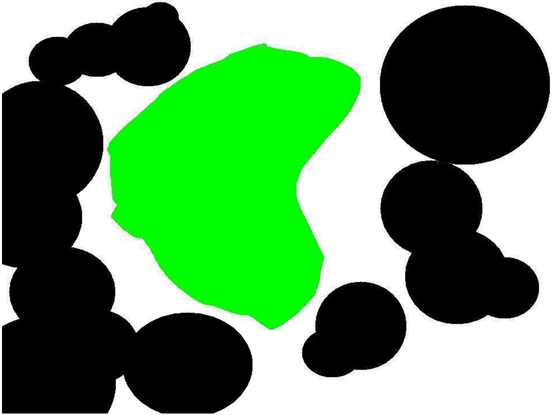

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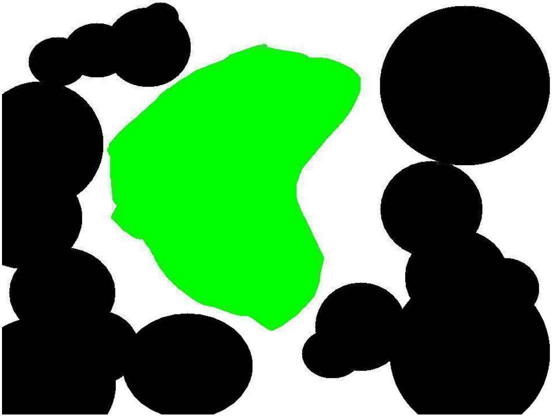

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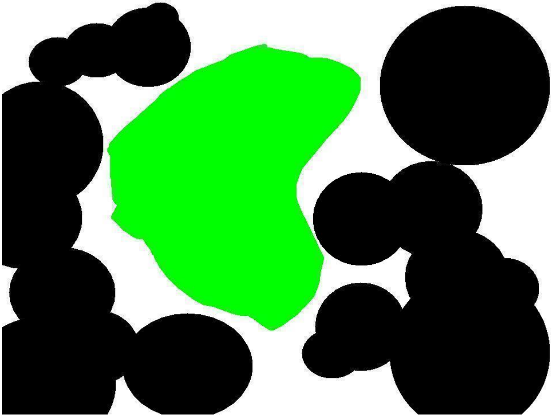

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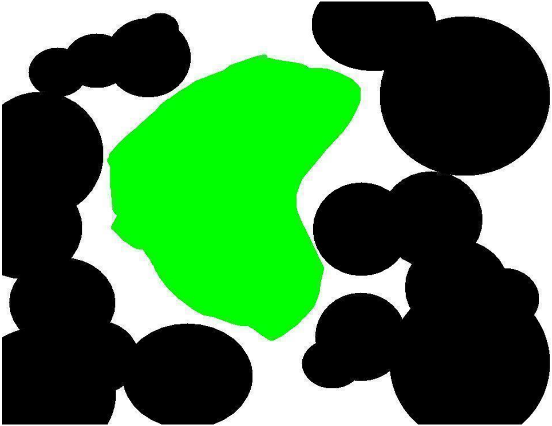

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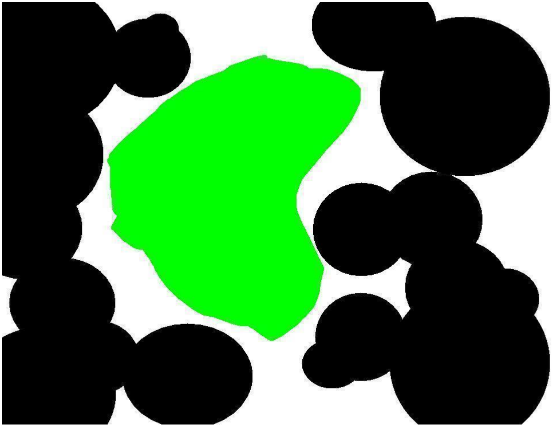

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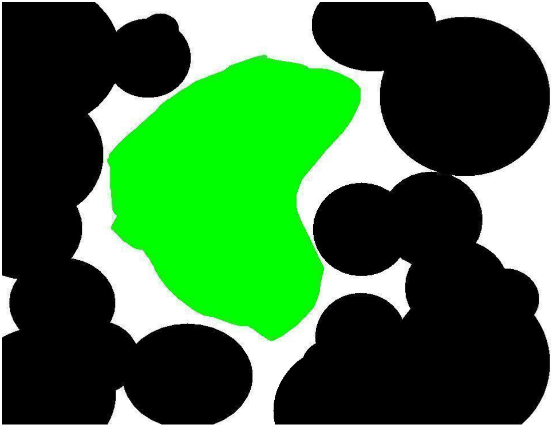

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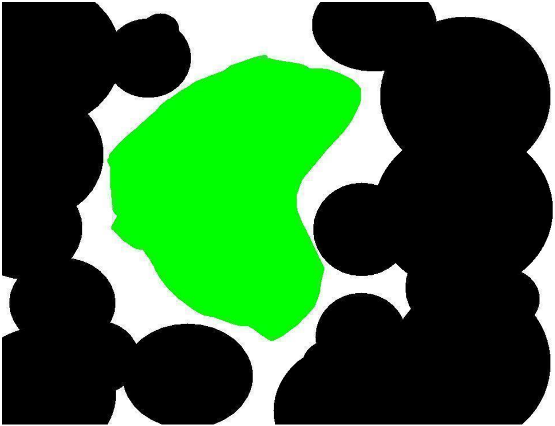

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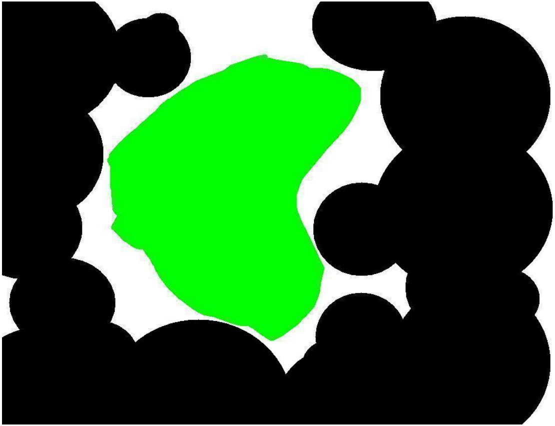

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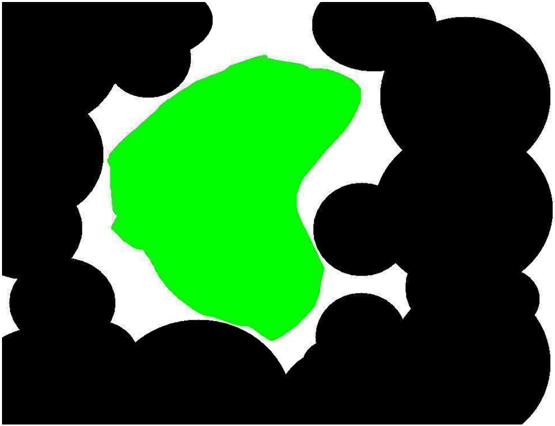

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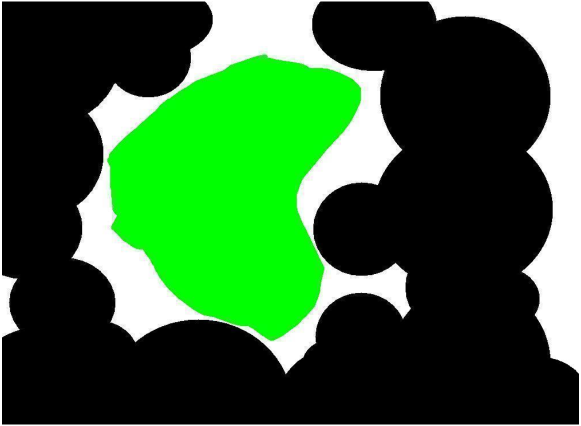

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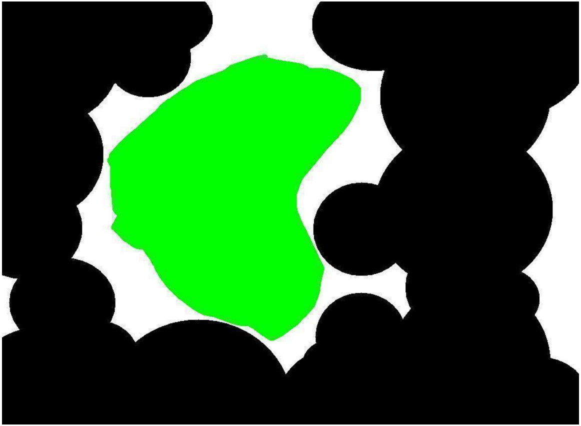

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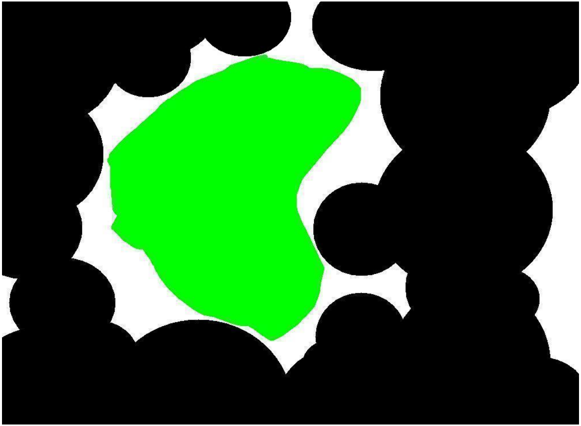

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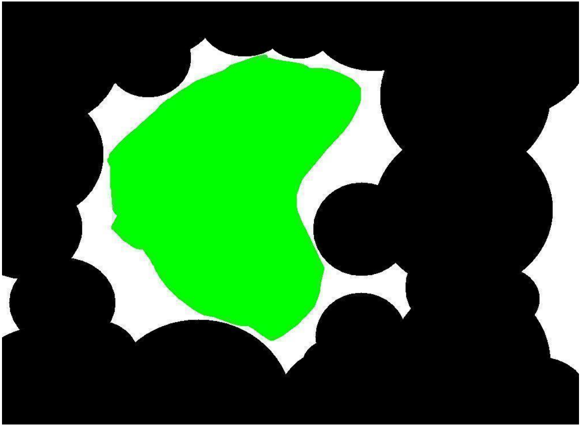

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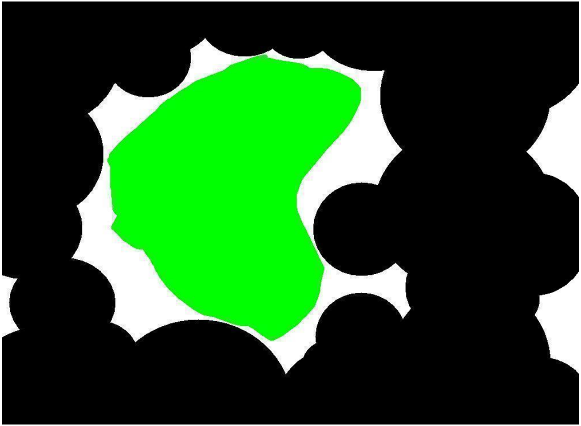

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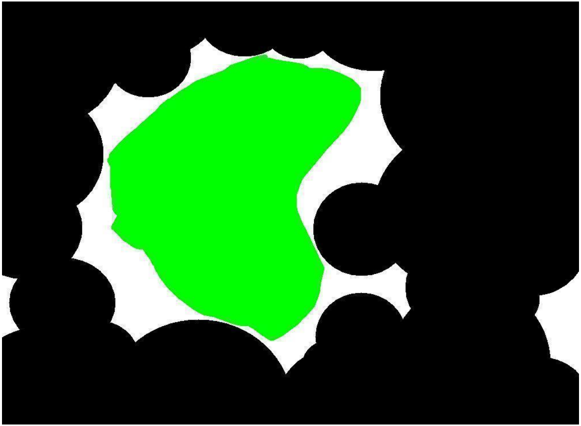

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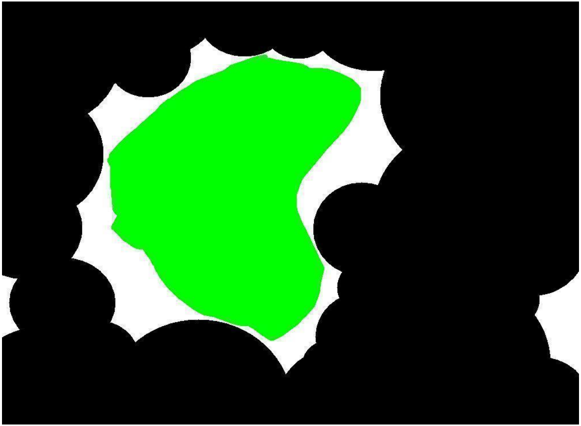

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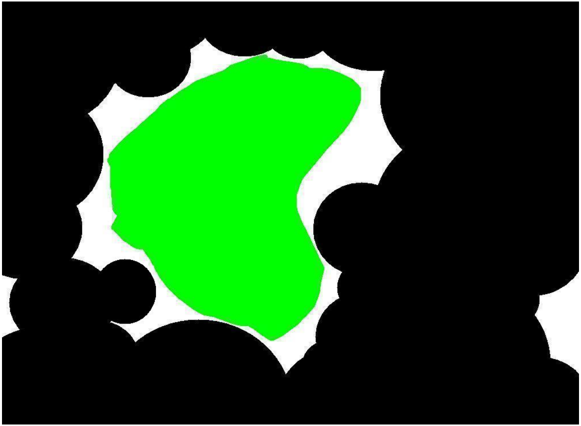

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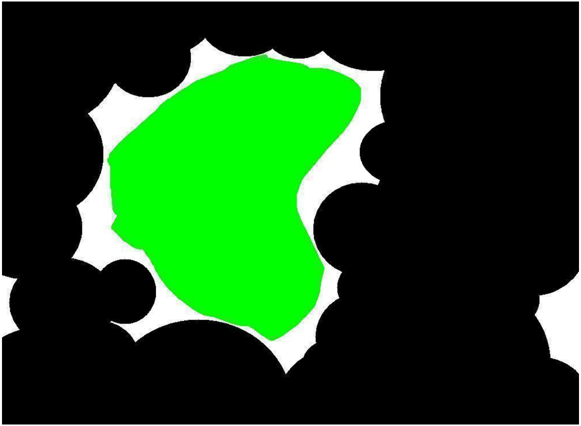

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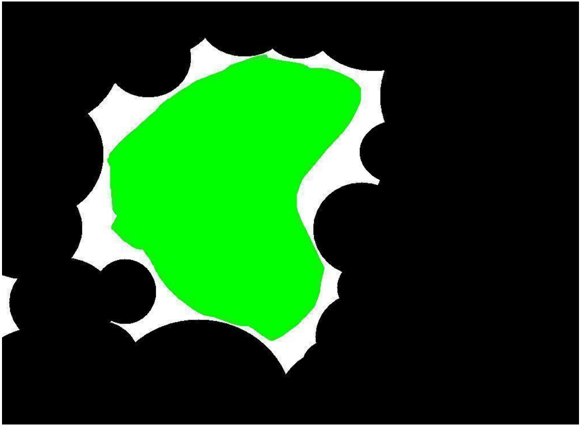

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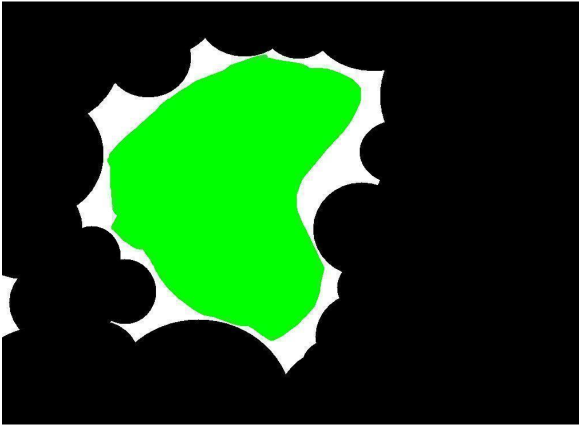

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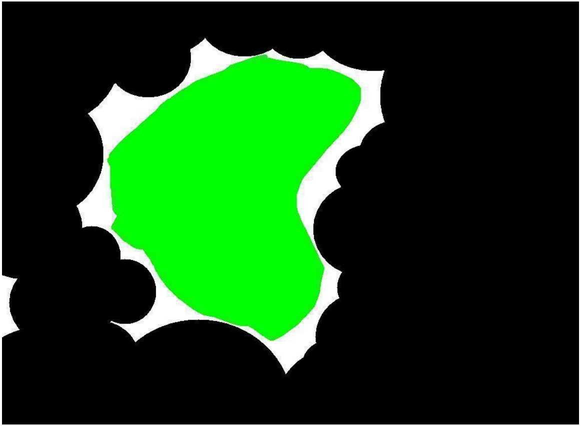

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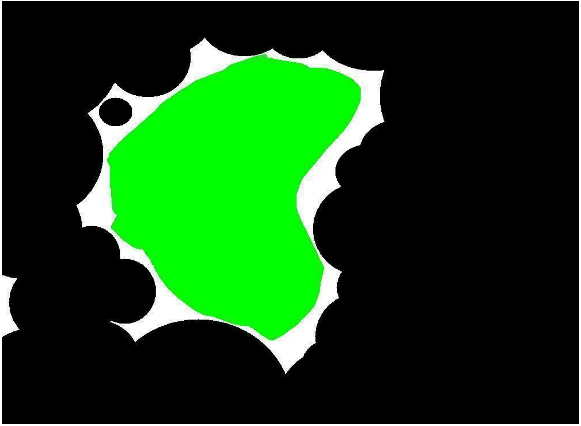

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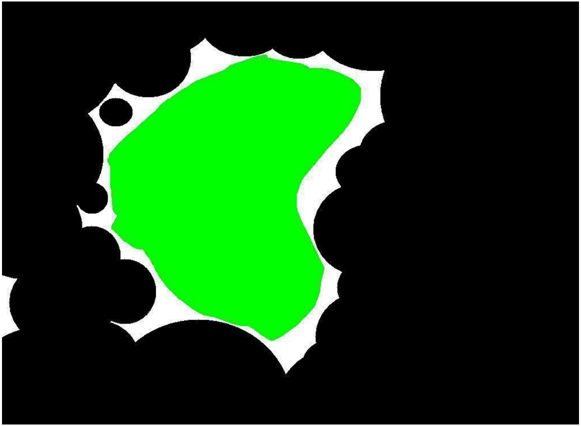

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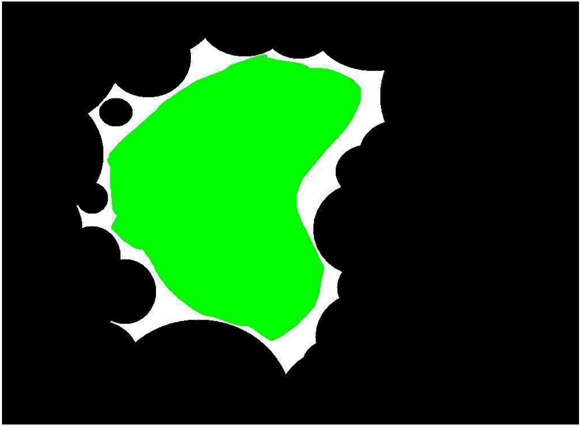

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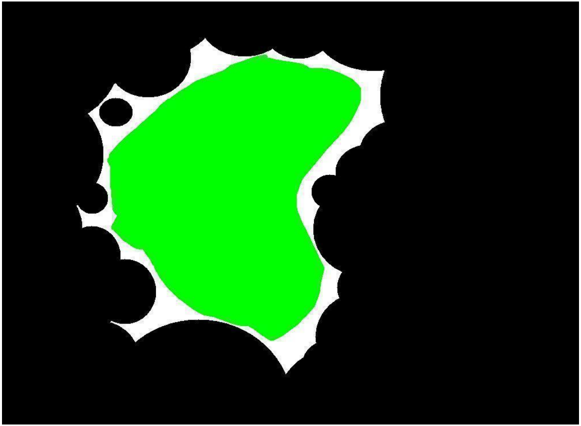

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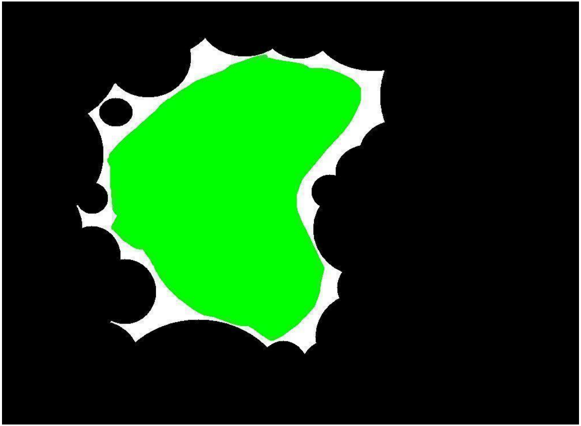

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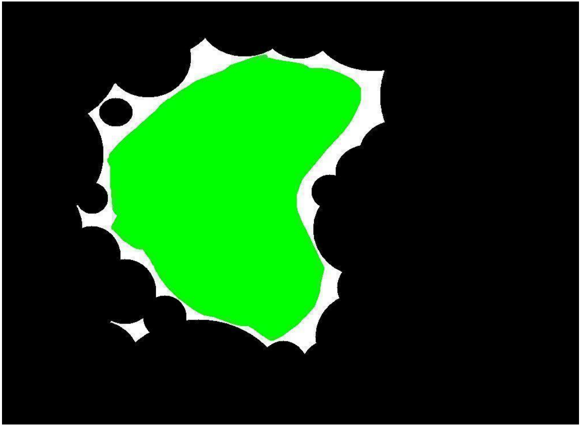

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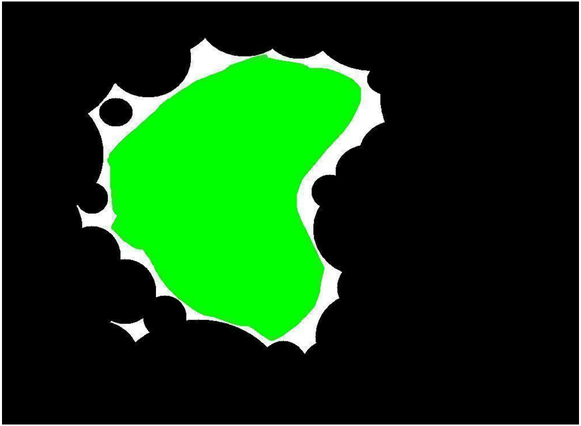

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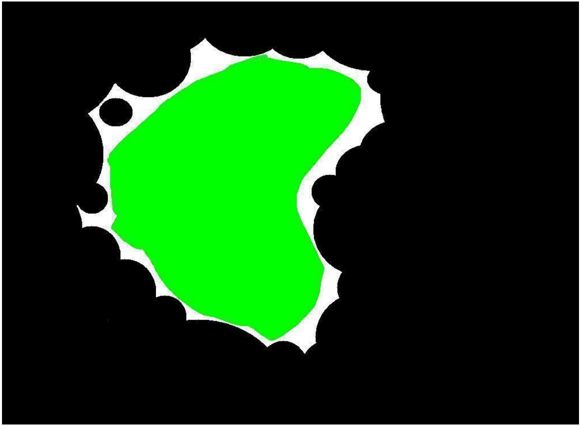

$S \neq \emptyset$ computable

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$$\Downarrow$$

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$S = \overline{\{x_i \mid i \in \mathbb{N}\}}$ for a computable sequence (x_i)

S semicomputable

S semicomputable

$$\Downarrow$$

S semicomputable

$$\Downarrow$$

S computable

S semicomputable and finite

S semicomputable and finite

$$\Downarrow$$

S computable

S semicomputable and locally Euclidean

S semicomputable and locally Euclidean

$$\Downarrow$$

S computable

$$S \subseteq \mathbb{R}^n$$

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$f : S \to S$ isometry

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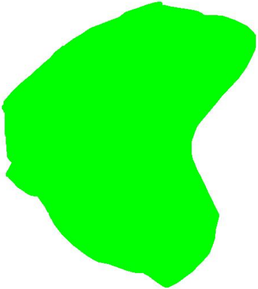
$$d(f(x), f(y)) = d(x, y) \text{ for all } x, y \in S$$

$S \subseteq \mathbb{R}^n$



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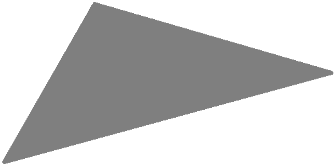

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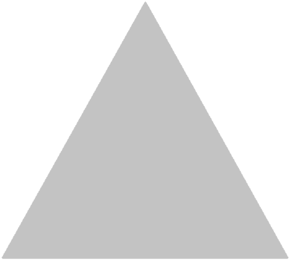

$S \subseteq \mathbb{R}^n$, $x \in S$

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x **rigid point** of S

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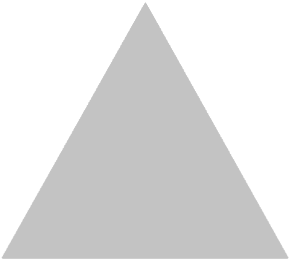
x **rigid point** of S

$f(x) = x$ for each isometry $f : S \to S$

$$S \subseteq \mathbb{R}^n$$



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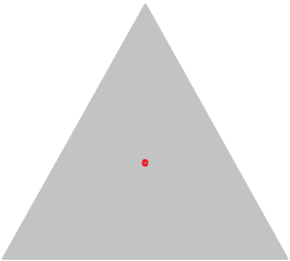

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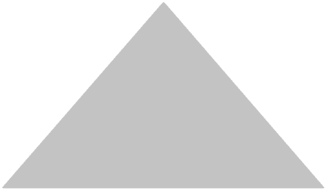

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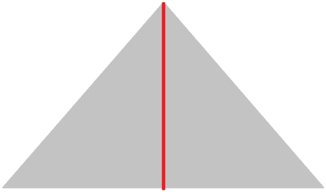

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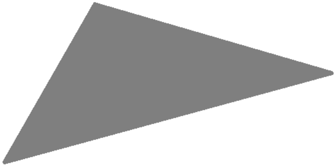
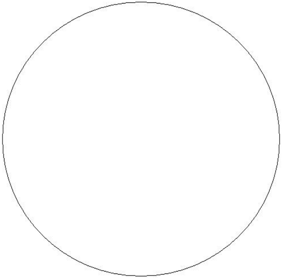
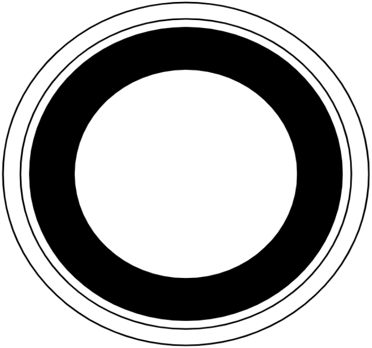
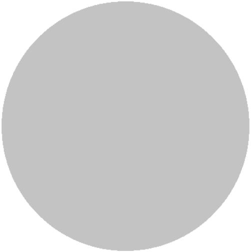
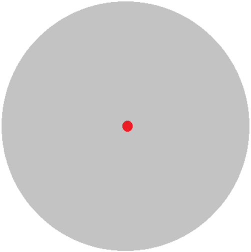
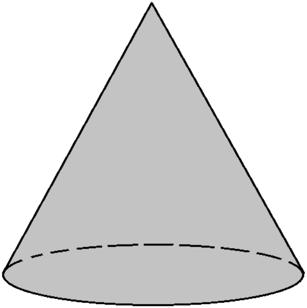


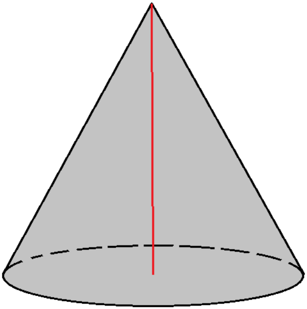












$S \neq \emptyset$ compact

$S \neq \emptyset$ compact and convex

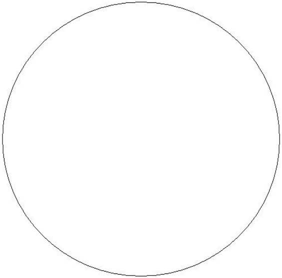
$S \neq \emptyset$ compact and convex

$\Rightarrow S$ has a rigid point

S computable

S computable

$\Rightarrow S$ has a computable rigid point



S computable

$\Rightarrow S$ has a computable rigid point

S computable and S has a rigid point

S computable and S has a rigid point

$\Rightarrow \mathsf{S}$ has a computable rigid point

S computable

S computable

- S has a rigid point $\not\Rightarrow$ S has a computable rigid point

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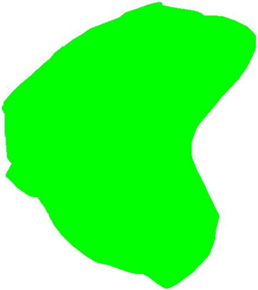
Theorem

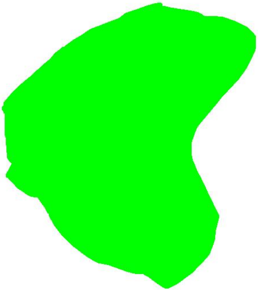
If $S \subseteq \mathbb{R}^n$ is a computable set, then the set of all rigid points of S is semicomputable.

$$S \subseteq \mathbb{R}^n$$



$S \subseteq \mathbb{R}^n$

$f : S \to S$ the only nontrivial isometry



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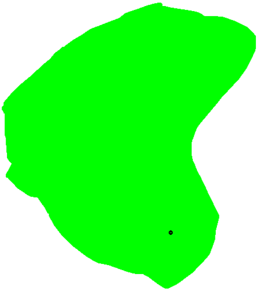

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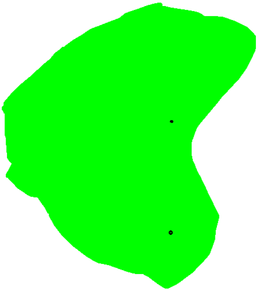

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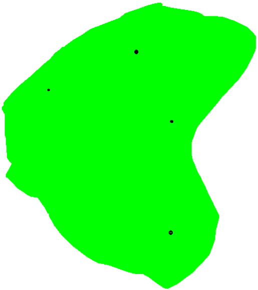

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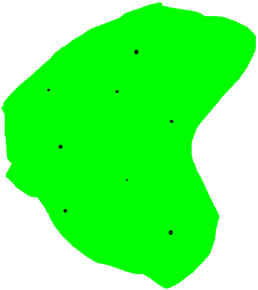

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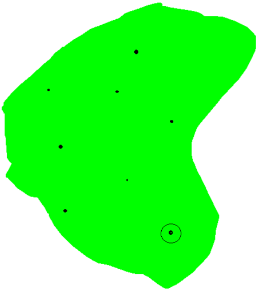

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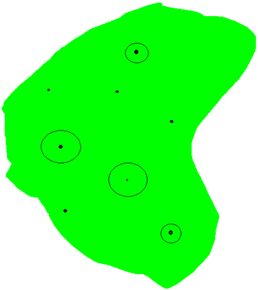

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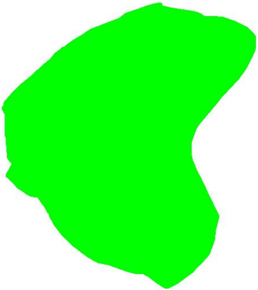

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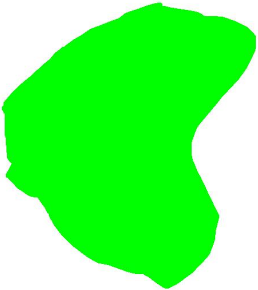
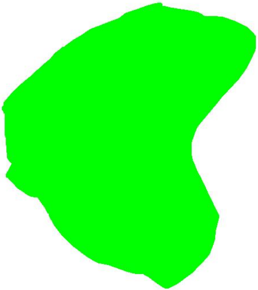


$$S \subseteq \mathbb{R}^n$$

$f_0, f_1, f_2, \ldots$ isometries dense in $\mathrm{Iso}(S)$



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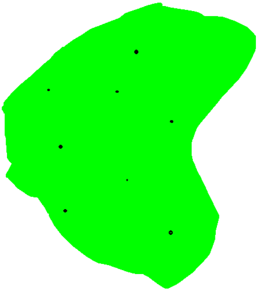

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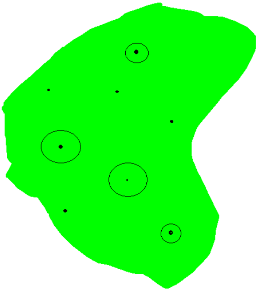

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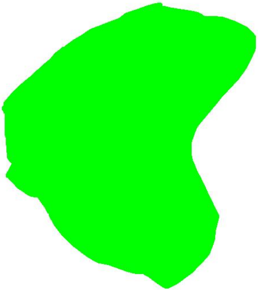

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$\mathrm{Iso}(S)$

$$S \subseteq \mathbb{R}^n$$

$\mathrm{Iso}(S) \subseteq C(S)$

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

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$$S^{\mathbb{N}}$$

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(α_i) computable dense sequence in S

$$(S^{\mathbb{N}}, \rho)$$

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

$$(S^{\mathbb{N}}, \rho, (a_n)) \text{ computable metric space}$$

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

$(S^{\mathbb{N}}, \rho, (a_n))$ computable metric space

$$\mathrm{Iso}(S) \to S^{\mathbb{N}}$$

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

$(S^{\mathbb{N}}, \rho, (a_n))$ computable metric space

$$\mathrm{Iso}(S) \to S^{\mathbb{N}}$$

$$f \mapsto (f(\alpha_i))$$

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

$(S^{\mathbb{N}}, \rho, (a_n))$ computable metric space

$$\mathrm{Iso}(S) \to S^{\mathbb{N}}$$

$$f \mapsto (f(\alpha_i))$$

Iso(S) is locally Euclidean

$$S \subseteq \mathbb{R}^n$$

(α_i) computable dense sequence in S

$(S^{\mathbb{N}}, \rho, (a_n))$ computable metric space

$$\mathrm{Iso}(S) \to S^{\mathbb{N}}$$

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$$S \subseteq \mathbb{R}^n$$

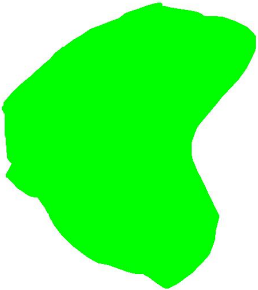
(α_i) computable dense sequence in S

$(S^{\mathbb{N}}, \rho, (a_n))$ computable metric space

$\mathrm{Iso}(S)$ is identified with a computable set in $(S^{\mathbb{N}}, \rho, (a_n))$

$$S \subseteq \mathbb{R}^n$$

$f_0, f_1, f_2, \ldots$ isometries dense in $\mathrm{Iso}(S)$



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