Computable categoricity and computable type

Computable type

S semicomputable $\Rightarrow$ S computable

$S \subseteq \mathbb{R}^n$ semicomputable

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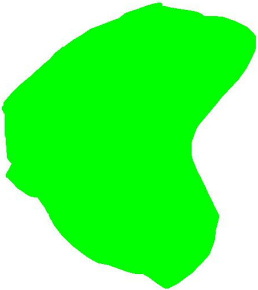

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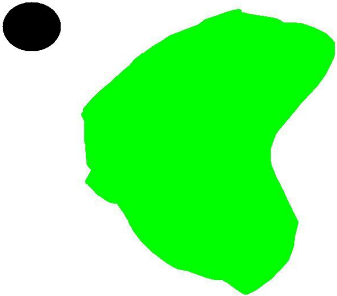

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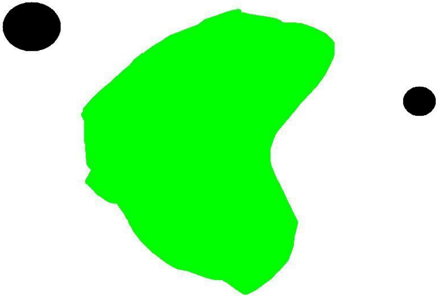

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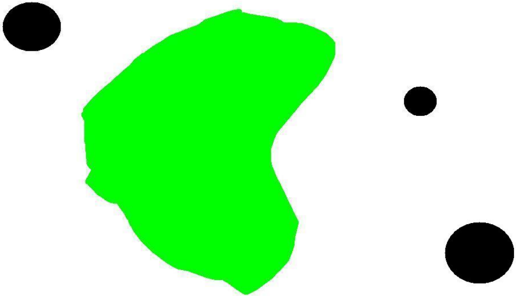

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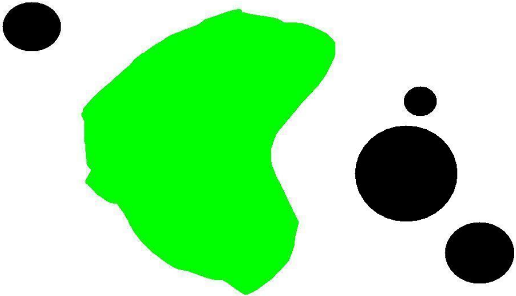

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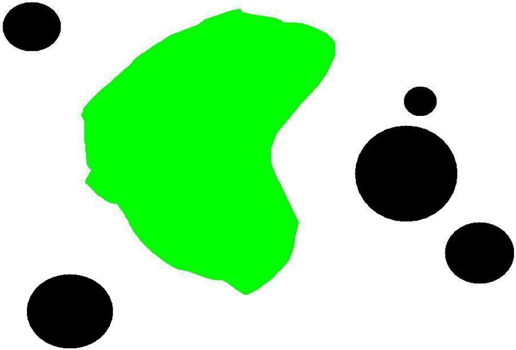

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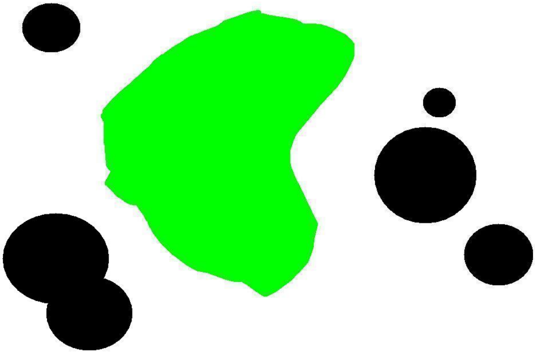

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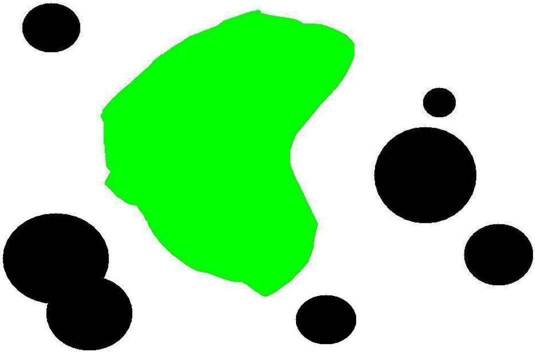

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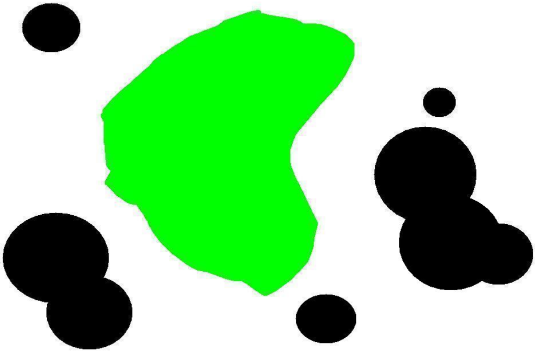

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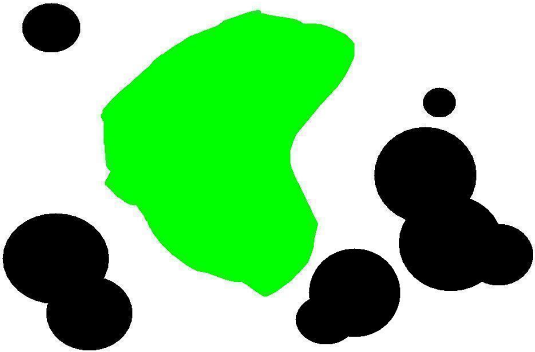

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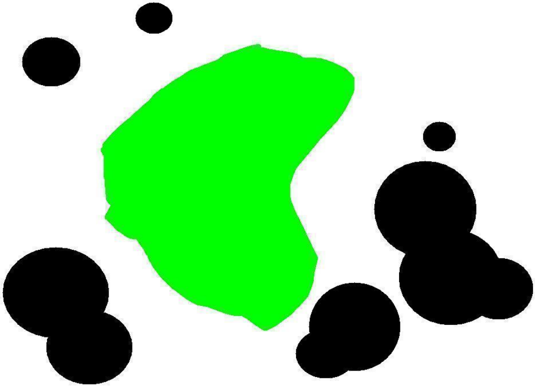

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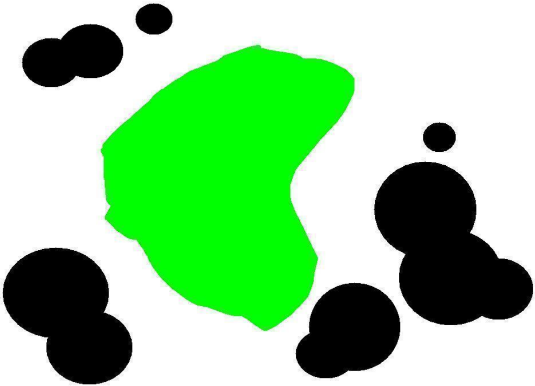

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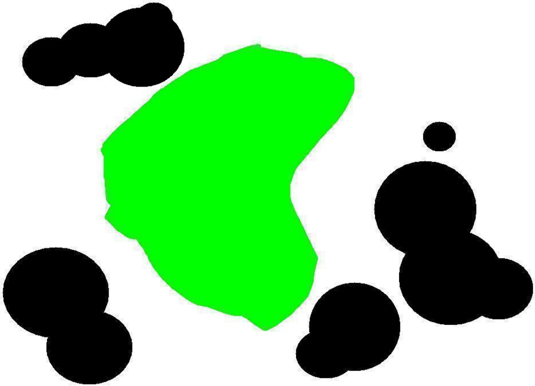

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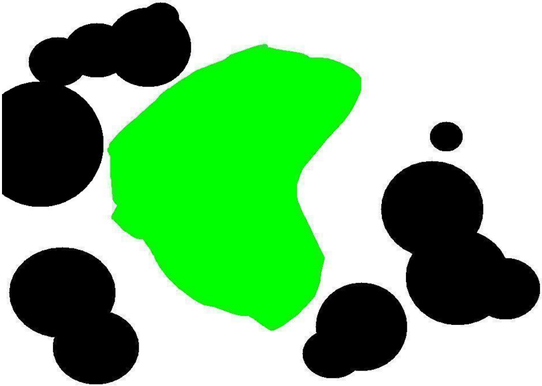

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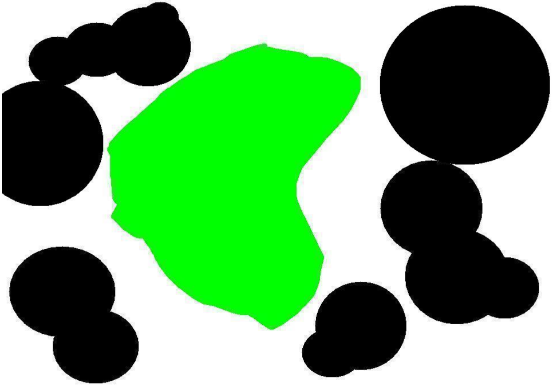

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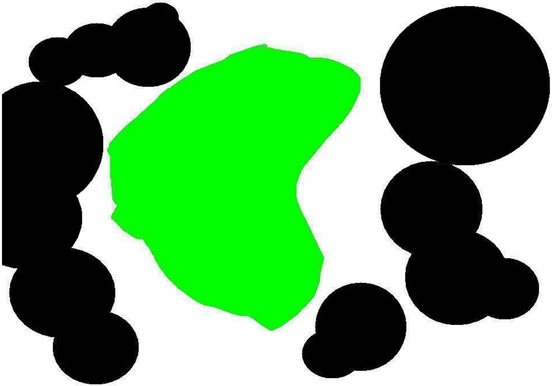

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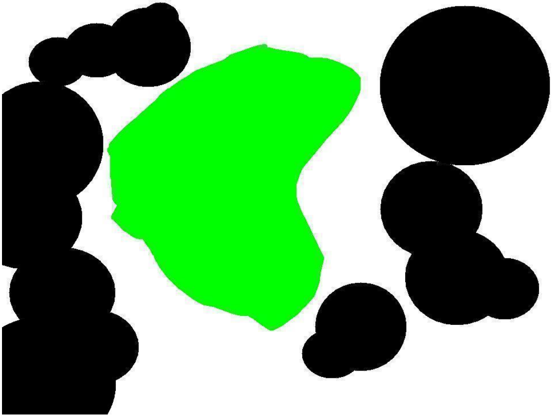

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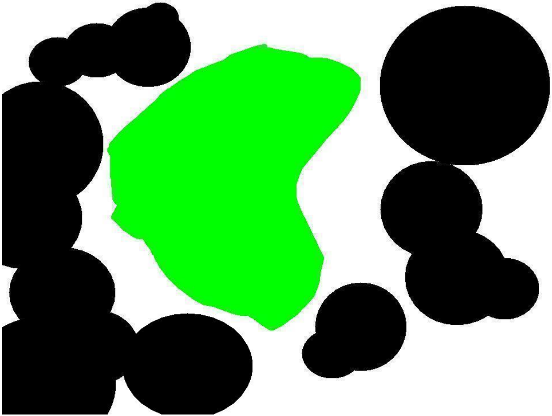

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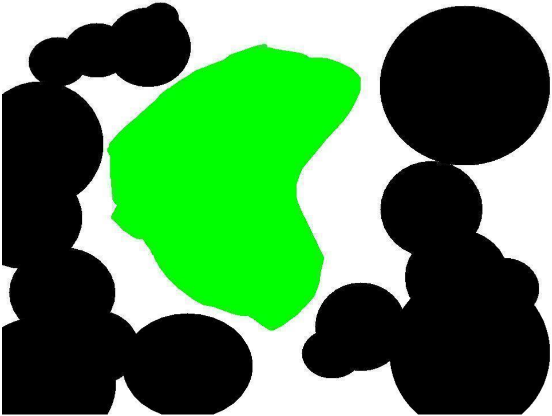

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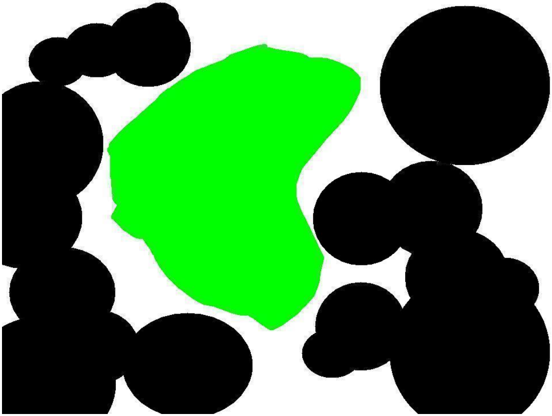

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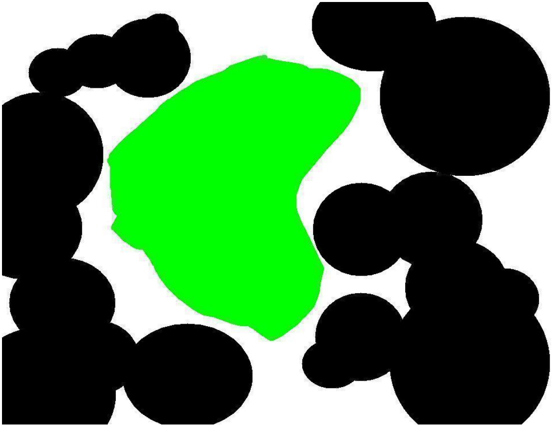

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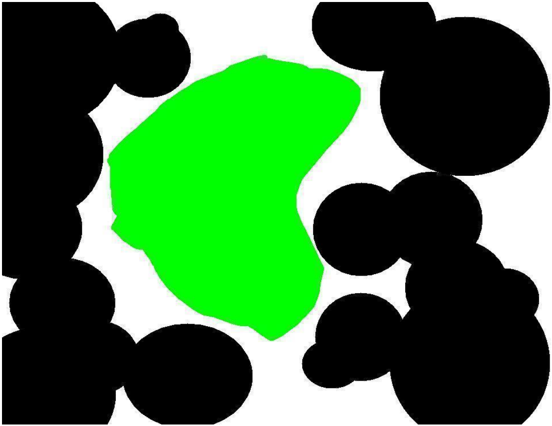

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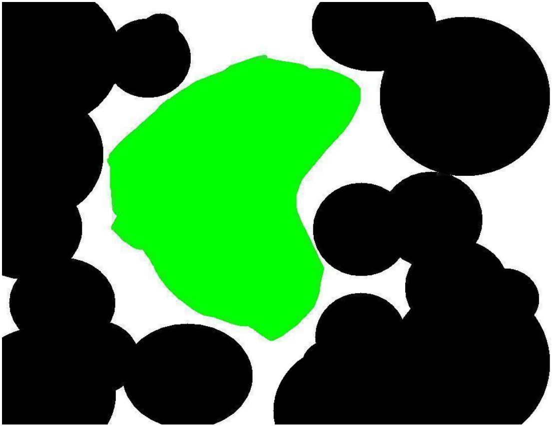

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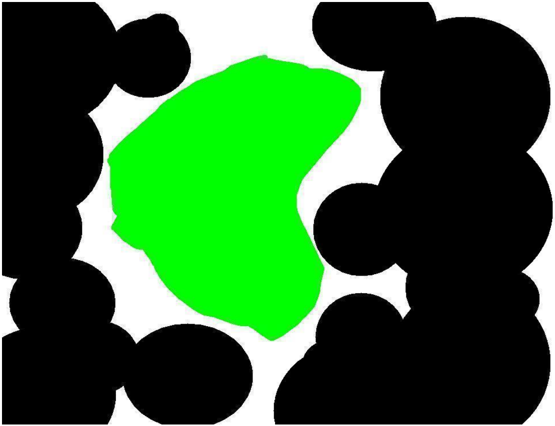

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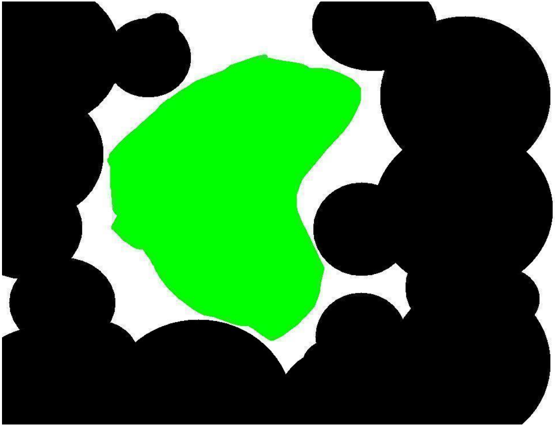

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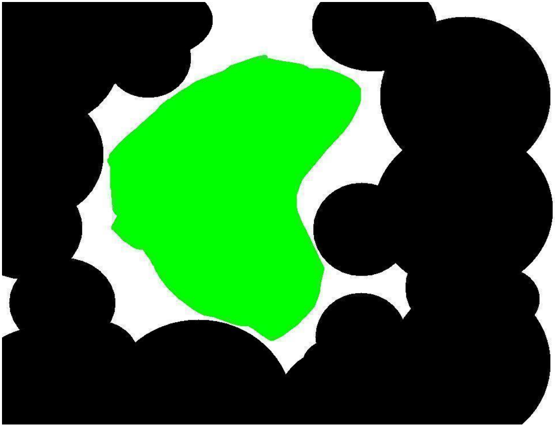

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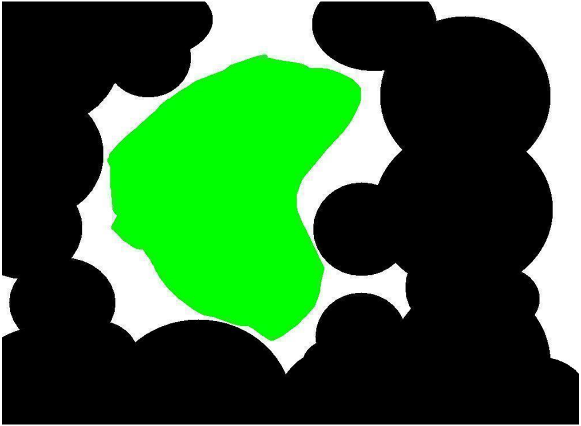

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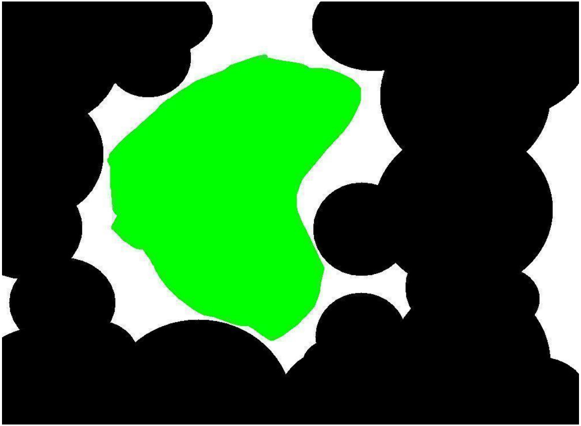

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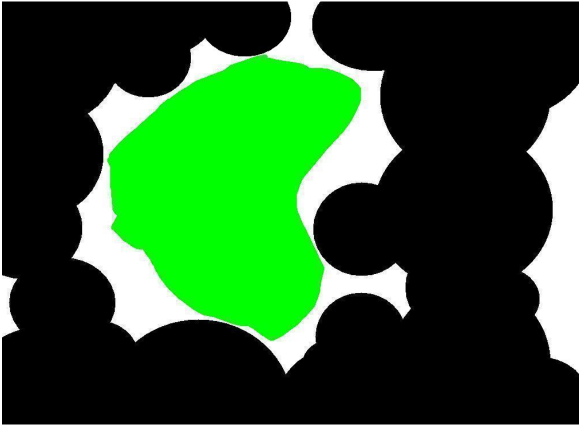

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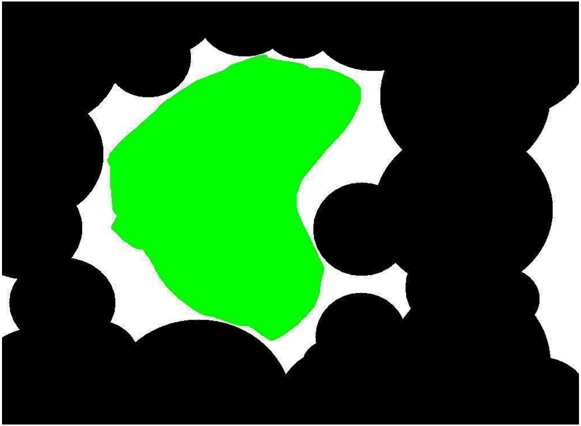

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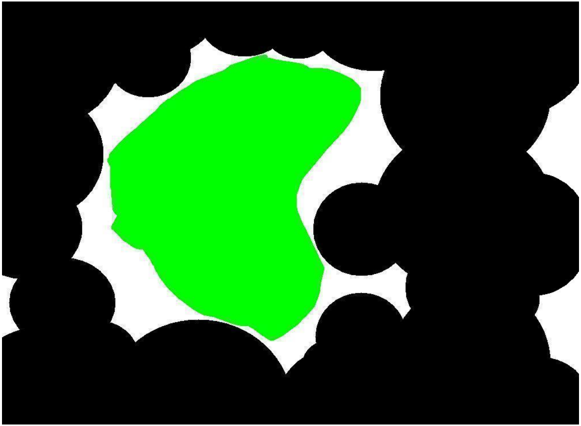

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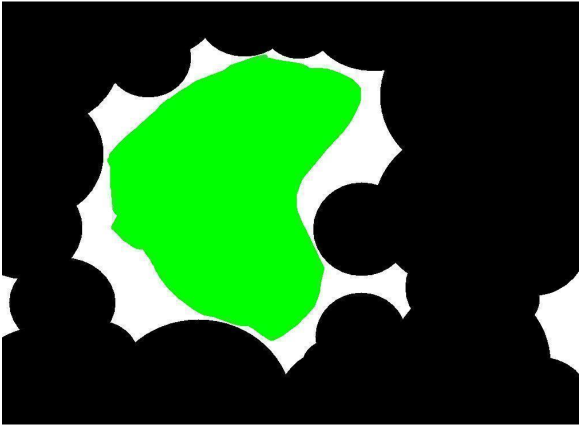

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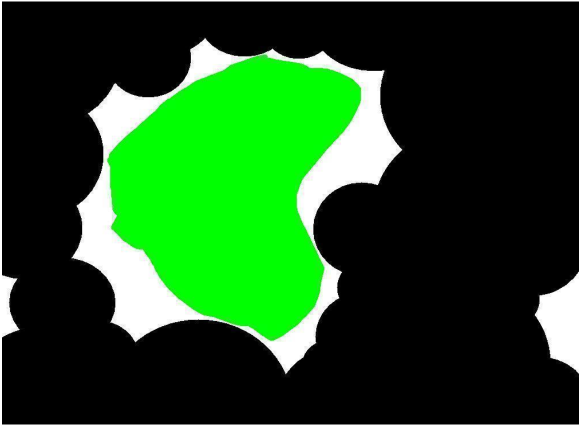

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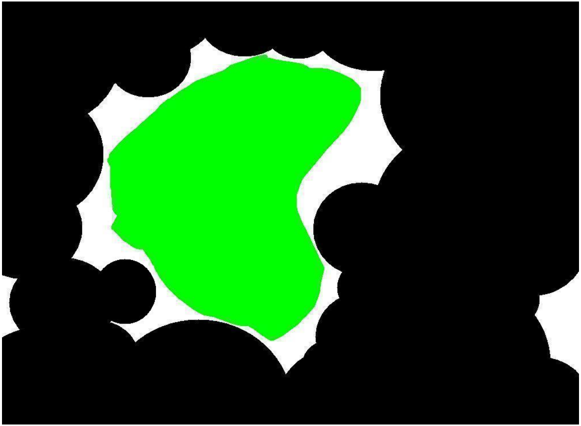

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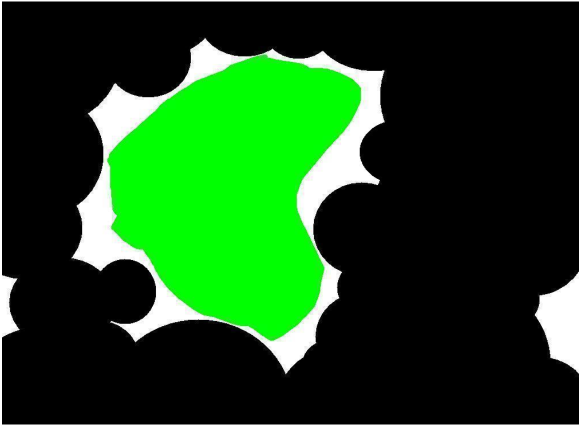

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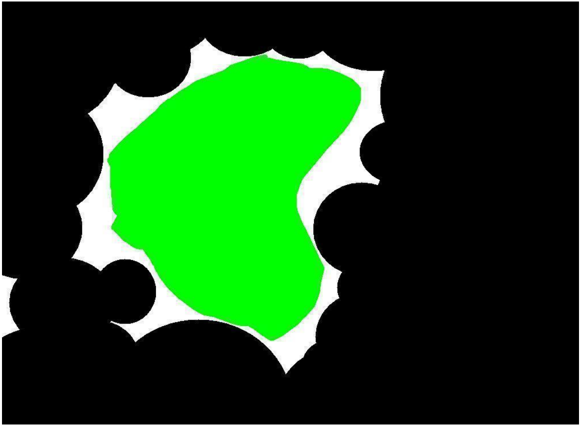

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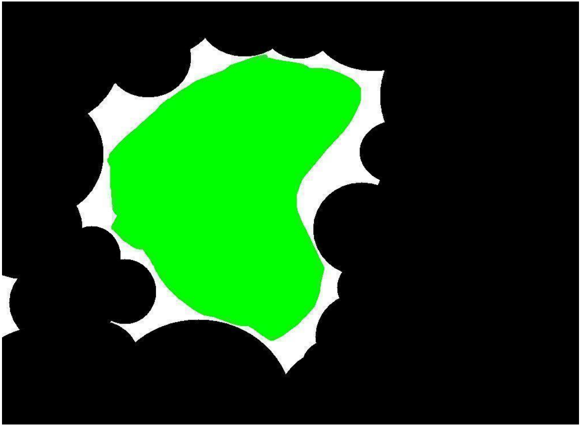

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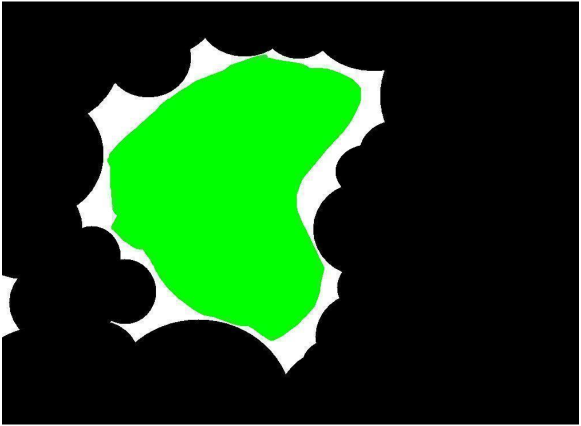

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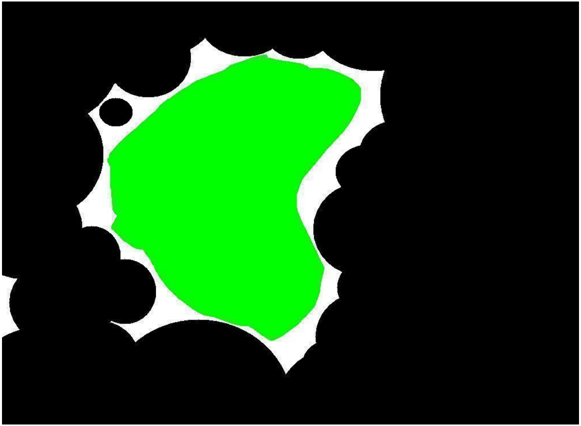

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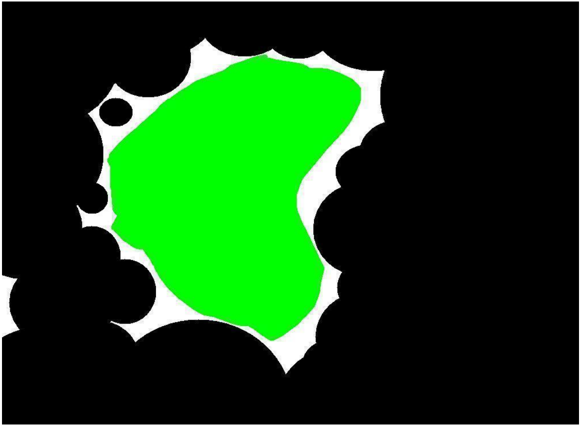

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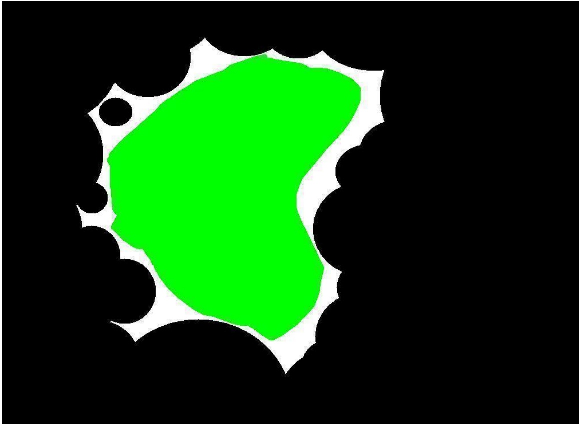

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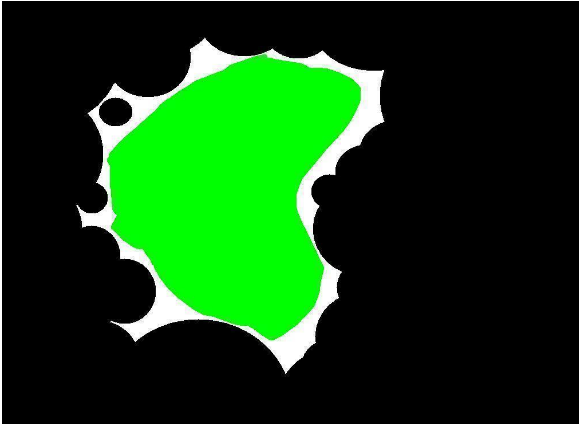

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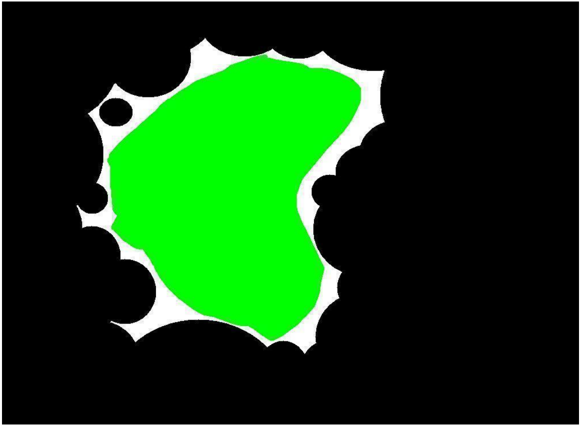

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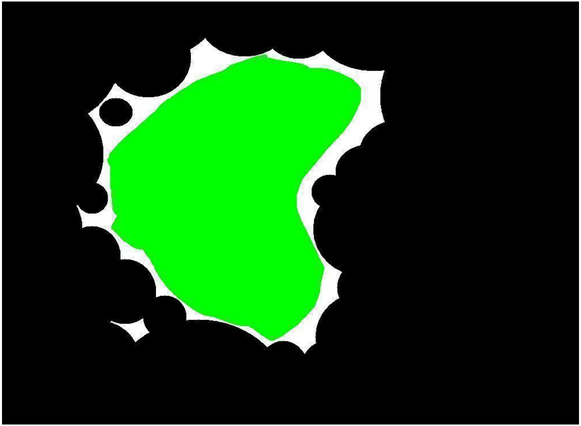

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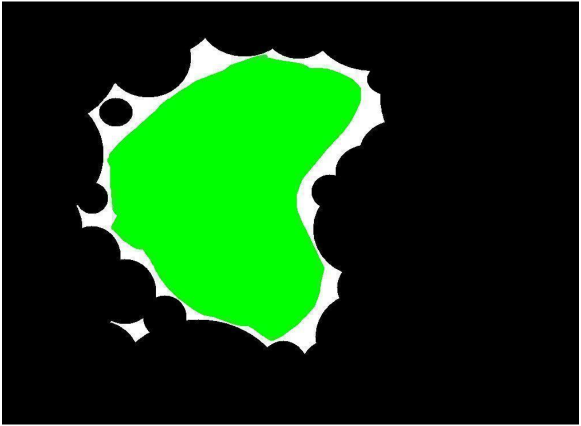

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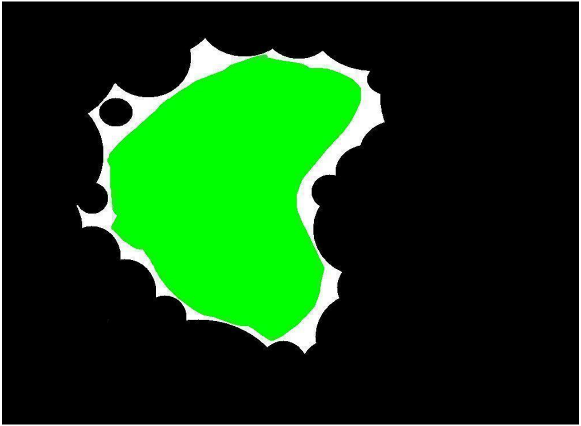

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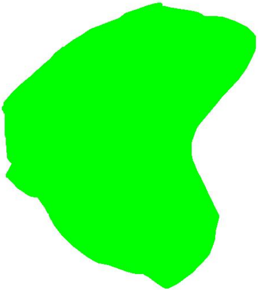

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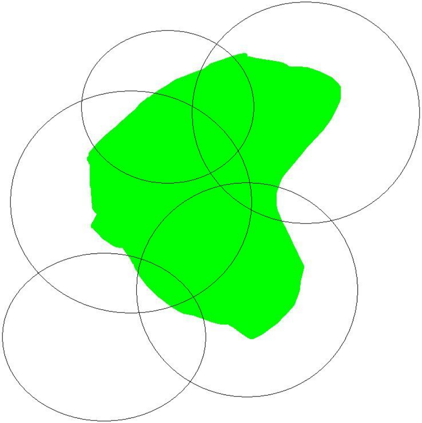

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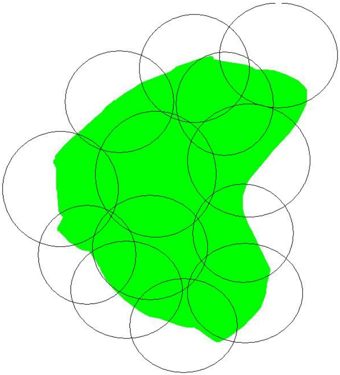

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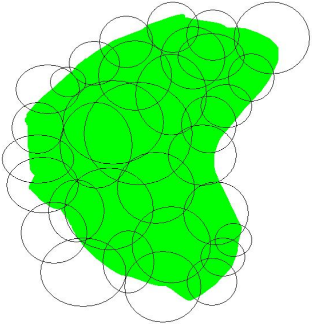

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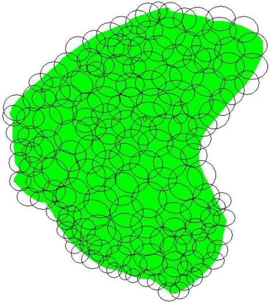

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conditions on S ???

S semicomputable $\Rightarrow$ S computable

S homeomorphic to $\mathbb{S}^1$

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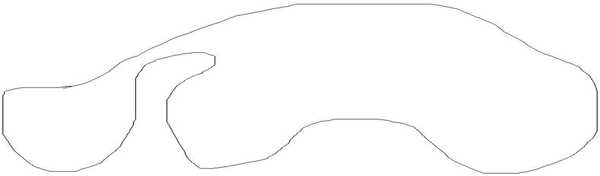

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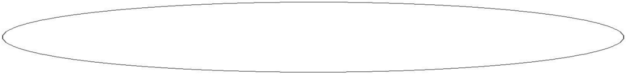

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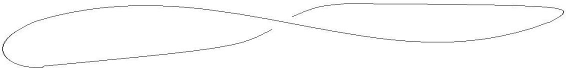

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S homeomorphic to $\mathbb{S}^m$

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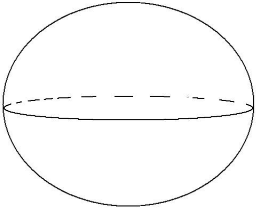

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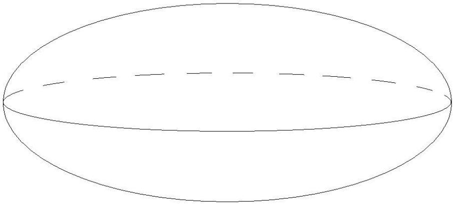

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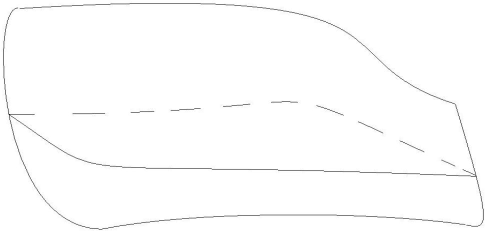

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S manifold

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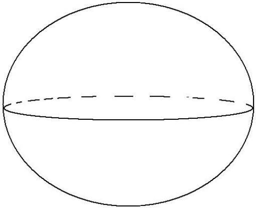

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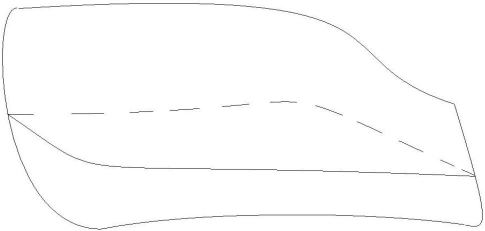

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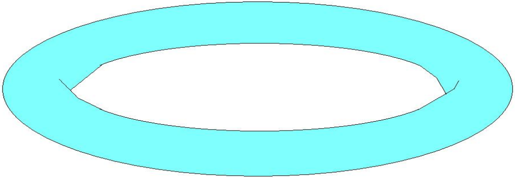

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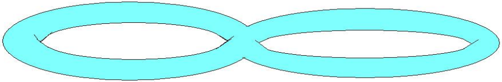

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$$\Updownarrow$$

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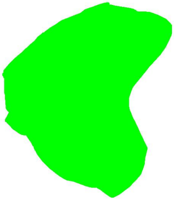

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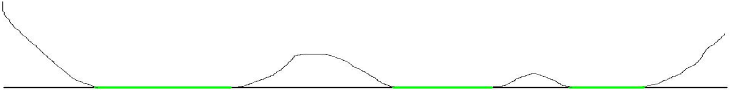

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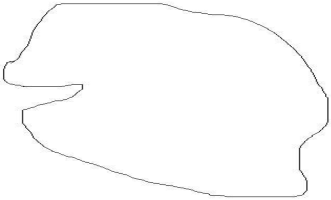
S manifold

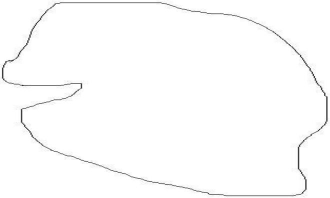
Computable categoricity

$$(X, d)$$

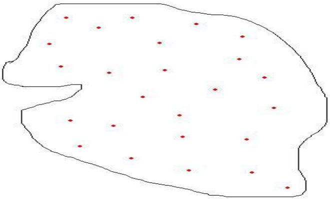
(X, d)



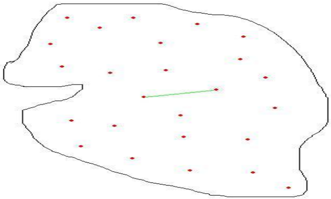
$$(X, d, \alpha)$$



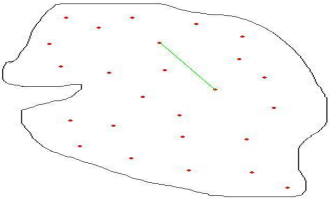
$$(X, d, \alpha)$$



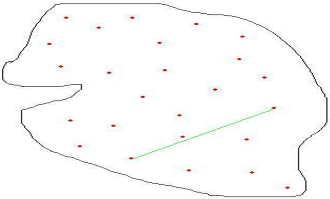
$$(X, d, \alpha)$$



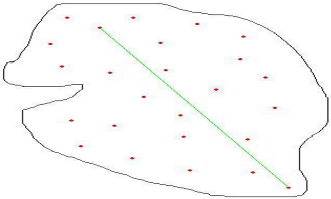
$$(X, d, \alpha)$$



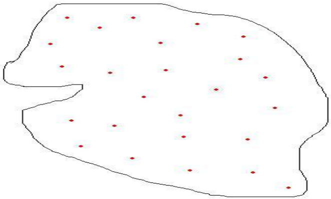
$$(X, d, \alpha)$$



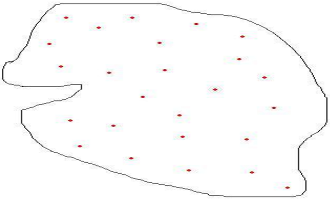
$$(X, d, \alpha)$$



$$(X, d, \alpha)$$

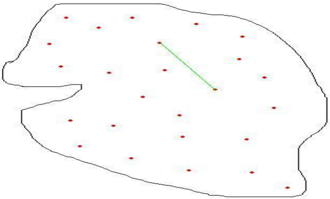


$$(X, d, \alpha)$$



$$(i, j) \mapsto d(\alpha_i, \alpha_j)$$

$$(X, d, \alpha)$$



$$(i, j) \mapsto d(\alpha_i, \alpha_j)$$

α effective separating sequence in (X, d)

α effective separating sequence in (X, d)

(X, d, α) computable metric space

$$(\mathbb{R}, d, \alpha)$$

$$(\mathbb{R}^n, d, \alpha)$$

$$(X, d, \alpha)$$

$$(X, d, \alpha)$$

$x_0 \in X$ computable

$$(X, d, \alpha)$$

$x_0 \in X$ computable

$$d(x_0, \alpha_{f(k)}) < 2^{-k}$$

$$(X, d, \alpha)$$

(x_i) computable

$$(X, d, \alpha)$$

(x_i) computable

$$d(x_i, \alpha_{f(i,k)}) < 2^{-k}$$

$$(X, d, \alpha)$$

$$(X, d, \alpha)$$

$S \subseteq X$

$$(X, d, \alpha)$$

$S \subseteq X$ computable

$$(X, d, \alpha)$$

$S \subseteq X$ computably enumerable

(X, d)

(X, d)

α, β effective separating sequences

$$\alpha \sim \beta$$

$$\alpha \sim \beta$$

α computable with respect to β

$$\alpha \sim \beta$$

$$(\mathbb{R}, d)$$

$(\mathbb{R}, d)$

(α_i) an effective enumeration of $\mathbb{Q}$

$(\mathbb{R}, d)$

(α_i) an effective enumeration of $\mathbb{Q}$

$\gamma \in \mathbb{R}$ incomputable

$(\mathbb{R}, d)$

(α_i) an effective enumeration of $\mathbb{Q}$

$\gamma \in \mathbb{R}$ incomputable

$\Longrightarrow (\alpha_i + \gamma)$ an effective separating sequence

$$(\mathbb{R}, d)$$

(α_i) an effective enumeration of $\mathbb{Q}$

$\gamma \in \mathbb{R}$ incomputable

$\Longrightarrow (\alpha_i + \gamma)$ an effective separating sequence

$$(\alpha_i) \nsim (\alpha_i + \gamma)$$

$$([0,1], d)$$

(X, d) effectively compact

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

$\alpha \sim \beta$ for all effective separating sequences α and β in (X, d)

(X, d) effectively compact

(X, d) compact

(X, d, α)

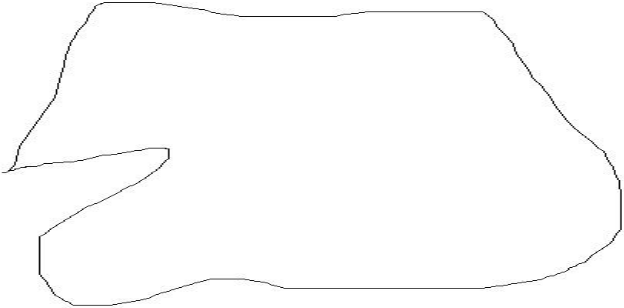
(X, d, α) effectively compact

(X, d, α) effectively compact

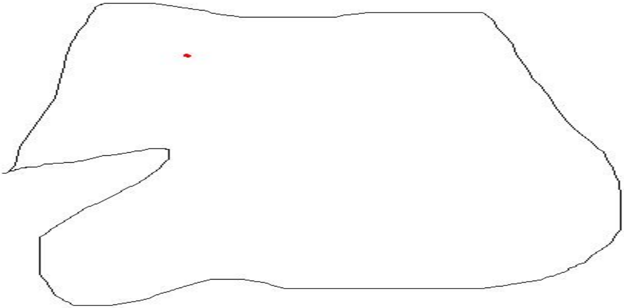
$$X = B(\alpha_0, 2^{-k}) \cup \cdots \cup B(\alpha_{f(k)}, 2^{-k}).$$

$f : \mathbb{N} \to \mathbb{N}$ computable

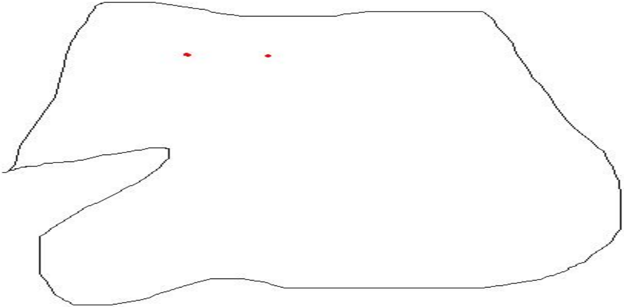
(X, d, α) effectively compact



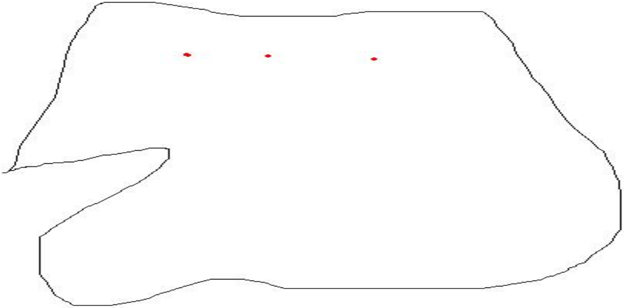
(X, d, α) effectively compact



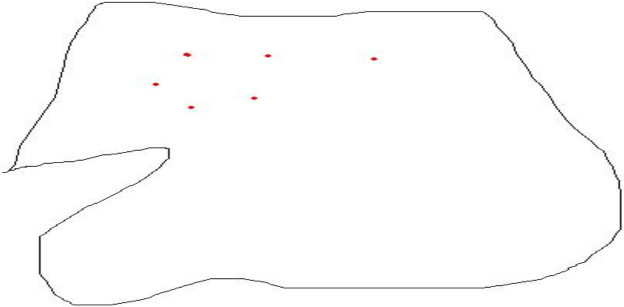
(X, d, α) effectively compact



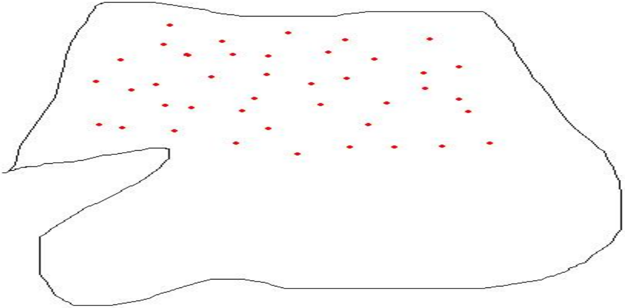
(X, d, α) effectively compact



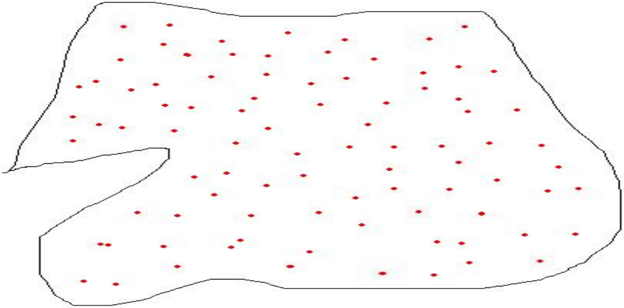
(X, d, α) effectively compact



(X, d, α) effectively compact



(X, d, α) effectively compact



(X, d) effectively compact

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

$\alpha \sim \beta$ for all effective separating sequences α and β in (X, d)

$$([0, \gamma], d)$$

$$([0, \gamma], d)$$

γ left computable, but incomputable

(S^1, d)

$$(X, d)$$

(α_i) effective separating sequence

$$(X, d)$$

(α_i) effective separating sequence

$f : X \to X$ surjective isometry

$$(X, d)$$

(α_i) effective separating sequence

$f : X \to X$ surjective isometry

$\implies (f(\alpha_i))$ effective separating sequence

(X, d) computably categorical

(X, d) computably categorical

(α_i), (β_i) effective separating sequences

(X, d) computably categorical

(α_i), (β_i) effective separating sequences

$\implies \exists\, f : X \to X$ surjective isometry

(X, d) computably categorical

(α_i), (β_i) effective separating sequences

$\implies \exists\, f : X \to X$ surjective isometry

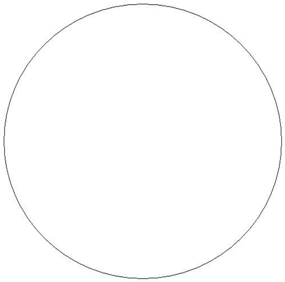
$$(\beta_i) \sim (f(\alpha_i))$$

(X, d) computably categorical

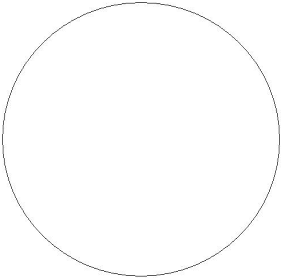
$(\mathbb{R}, d)$ computably categorical

$(\mathbb{R}^n, d)$ computably categorical

(S^1, d)



(S^1, d) computably categorical



(X, d) effectively compact

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

(X, d) computably categorical

$$([0, \gamma], d)$$

$$([0,\gamma], d)$$

γ left computable, but incomputable

Cantor space

Cantor space

Urysohn space

Cantor space

Urysohn space

every separable Hilbert space

Cantor space

Urysohn space

every separable Hilbert space

$C[0, 1]$ not computably categorical

Cantor space

Urysohn space

every separable Hilbert space

$C[0,1]$ not computably categorical

l^p computably categorical $\Leftrightarrow p = 2$

(X, d) compact

(X, d) effectively compact

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

$$\Downarrow$$

(X, d) effectively compact

and there exist only finitely many isometries $f : X \to X$

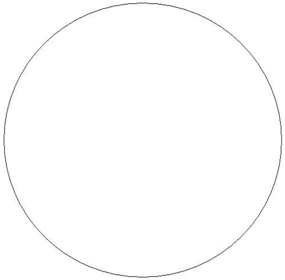
$$\Downarrow$$

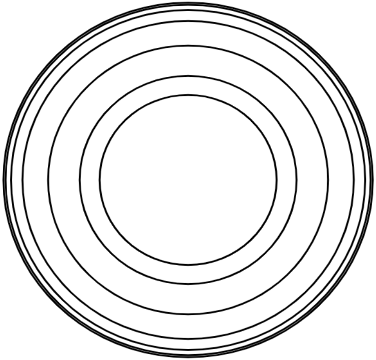
(X, d) computably categorical

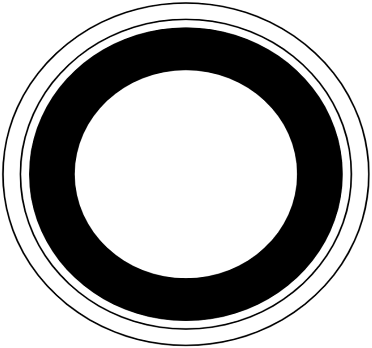
(X, d) effectively compact

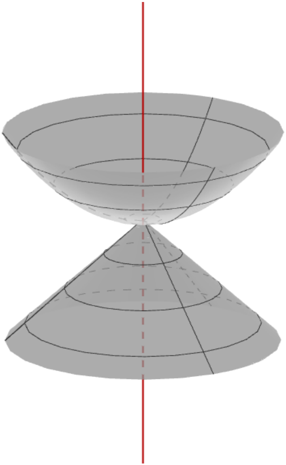
(X, d) effectively compact

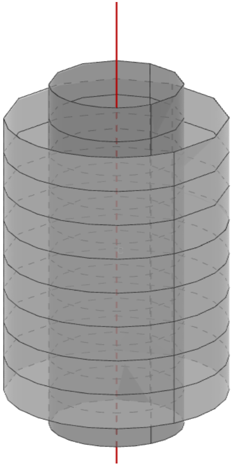
$\textsc{Iso}(X, d)$ infinite

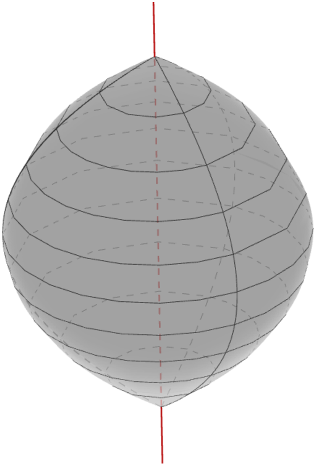


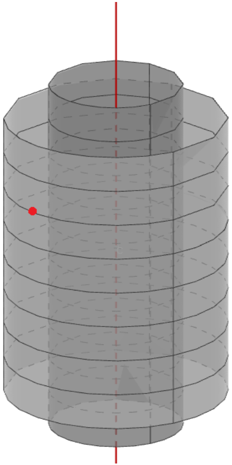


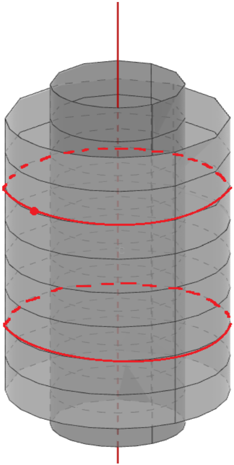












every effectively compact subspace of $\mathbb{R}^2$ and $\mathbb{R}^3$ is computably categorical is every effectively compact subspace of $\mathbb{R}^n$ computably categorical?

Theorem

Let (X, d) be an effectively compact metric space

Theorem

Let (X, d) *be an effectively compact metric space such that* $\mathrm{Iso}(X, d)$ *is a manifold.*

Theorem

Let (X, d) be an effectively compact metric space such that $\mathrm{Iso}(X, d)$ is a manifold.

$$d_\infty(f, g) = \sup\{d(f(x), g(x)) \mid x \in X\}$$

Theorem

Let (X, d) *be an effectively compact metric space such that* $\mathrm{Iso}(X, d)$ *is a manifold.*

Theorem

Let (X, d) *be an effectively compact metric space such that* $\mathrm{Iso}(X, d)$ *is a manifold. Then* (X, d) *is computably categorical.*

X effectively compact subspace of $\mathbb{R}^n$

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

$\mathrm{Iso}(X)$ manifold

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

$\mathrm{Iso}(X)$ manifold

$$\Downarrow$$

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

$\mathrm{Iso}(X)$ manifold

$$\Downarrow$$

(X, d) computably categorical

Theorem

$$(X, d) \textit{ effectively compact metric space}$$

$$\mathrm{Iso}(X, d) \textit{ manifold}$$

$$\Downarrow$$

$$(X, d) \textit{ computably categorical}$$

(X, d, α) effectively compact

(X, d, α) effectively compact

$$X^{\mathbb{N}}$$

(X, d, α) effectively compact

$(X^{\mathbb{N}}, \rho)$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho)$$

$$\rho((x_i), (y_i)) = \sup\{\tfrac{1}{2^i} d(x_i, y_i) \mid i \in \mathbb{N}\}$$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho)$$

$$\rho((x_i), (y_i)) = \sup\{\tfrac{1}{2^i} d(x_i, y_i) \mid i \in \mathbb{N}\}$$

$$(X^{\mathbb{N}}, \rho, (a_n))$$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho, (a_n))$$

(X, d, α) effectively compact

$(X^{\mathbb{N}}, \rho, (a_n))$

(x_i) computable point in $(X^{\mathbb{N}}, \rho, (a_n))$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho, (a_n))$$

(x_i) computable point in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Updownarrow$$

(X, d, α) effectively compact

$(X^{\mathbb{N}}, \rho, (a_n))$

(x_i) computable point in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Updownarrow$$

(x_i) computable sequence in (X, d, α)

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho, (a_n))$$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho, (a_n))$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

(X, d, α) effectively compact

$$(X^{\mathbb{N}}, \rho, (a_n))$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$\gamma \sim_{iso} \beta$ means $d(\gamma_i, \gamma_j) = d(\beta_i, \beta_j)$ for all $i, j \in \mathbb{N}$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Downarrow$$

$\mathrm{Im}(\Gamma)$ semicomputable in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Downarrow$$

$\mathrm{Im}(\Gamma)$ computable in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Downarrow$$

$\mathrm{Im}(\Gamma)$ computable in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Downarrow$$

$\exists f$ such that $f \circ \beta$ computable point in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Gamma : \mathrm{Iso}(X, d) \to X^{\mathbb{N}}$$

$$\Gamma(f) = (f(\beta_i))$$

$$\mathrm{Im}(\Gamma) = \{\gamma \in X^{\mathbb{N}} \mid \gamma \sim_{iso} \beta\}$$

$$\Downarrow$$

$\mathrm{Im}(\Gamma)$ computable in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Downarrow$$

$\exists f$ such that $f \circ \beta$ computable point in $(X^{\mathbb{N}}, \rho, (a_n))$

$$\Rightarrow f \circ \beta \sim \alpha$$

Theorem

(X, d) *effectively compact metric space*

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *manifold*

Theorem

(X, d) effectively compact metric space

$\mathrm{Iso}(X, d)$ manifold

$$\Downarrow$$

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *manifold*

$$\Downarrow$$

(X, d) *computably categorical*

X effectively compact subspace of $\mathbb{R}^n$

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

X effectively compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

$\mathrm{Iso}(X)$ manifold

X compact subspace of $\mathbb{R}^n$

X compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

X compact subspace of $\mathbb{R}^n$

$$\Downarrow$$

$\textsc{Iso}(X)$ manifold

X compact subspace of $\mathbb{R}^n$

X compact subspace of $\mathbb{R}^n$

$$\mathrm{Iso}(X, d) \to O(n) \times \mathbb{R}^n$$

X compact subspace of $\mathbb{R}^n$

$$\mathrm{Iso}(X, d) \to O(n) \times \mathbb{R}^n$$

$$f(x) = Ax + b$$

X compact subspace of $\mathbb{R}^n$

$$\textsc{Iso}(X, d) \to O(n) \times \mathbb{R}^n$$

X compact subspace of $\mathbb{R}^n$

$$\mathrm{Iso}(X, d) \to O(n) \times \mathbb{R}^n$$

$$(A, b) * (C, d) = (AC, Ad + b)$$

Theorem

Suppose G is a Lie Group and H is a subgroup of G that is also a closed subset. Then H is a Lie group.

X compact subspace of $\mathbb{R}^n$

$$\mathrm{Iso}(X, d) \to O(n) \times \mathbb{R}^n$$

Theorem

(X, d) *effectively compact metric space*

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *manifold*

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *finite*

Theorem

$$(X, d) \textit{ effectively compact metric space}$$

$$\mathrm{Iso}(X, d) \textit{ finite}$$

$$\Downarrow$$

Theorem

$$(X, d) \textit{ effectively compact metric space}$$

$$\mathrm{Iso}(X, d) \textit{ finite}$$

$$\Downarrow$$

$$\alpha \sim \beta \textit{ for all eff. sep. seq. } \alpha \textit{ and } \beta \textit{ in } (X, d)$$

Theorem

(X, d) *effectively compact metric space*

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *manifold*

Theorem

$$(X, d) \text{ effectively compact metric space}$$

$$\mathrm{Iso}(X, d) \text{ manifold}$$

$$\Downarrow$$

Theorem

$$(X, d) \textit{ effectively compact metric space}$$

$$\mathrm{Iso}(X, d) \textit{ manifold}$$

$$\Downarrow$$

$$\exists f \quad \textit{such that } f \circ \alpha \sim \beta$$

Theorem

(X, d) *effectively compact metric space*

$\mathrm{Iso}(X, d)$ *manifold*

$$\Downarrow$$

$\forall \varepsilon > 0 \; \exists f \;\; d_\infty(f, id_X) < \varepsilon$ *and* $f \circ \alpha \sim \beta$